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THE UNIVERSITY OF ALBERTA

AN EXPERIENCE ORIENTED MASTERY LEARNING STRATEGY IN NINTH GRADE ALGEBRA - AN EXPERIMENTAL STUDY

by

© DALE R. DROST

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

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FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "An Experience Oriented Mastery Learning Strategy in Ninth Grade Algebra - An Experimental Study", submitted by Dale R. Drost, in partial fulfilment of the requirements for the degree of Master of Education.

Thomas E. Kieren
Supervisor

Date Nov. 18, 1974

ABSTRACT

The purpose of the study was to develop a mastery learning strategy and to investigate the effects of the strategy on students and their learning. Possible tenets of a mastery learning theory and characteristics of mastery learning strategies were identified by reviewing the related literature. Based on these characteristics, a unit of content was selected and basic learning task objectives defined for that unit. Formative tests and corrective experiences were developed for use in a mastery strategy.

Six grade nine classes in one school were given various sequences of the mastery treatment and a nonmastery treatment, over a 4 week period. Mastery of the content was defined for each basic learning task, each subunit, and the entire unit.

The results indicated that a higher proportion of students who received only the mastery treatment attained mastery at all levels than did the proportion of students who received only the nonmastery treatment. Both of these groups attained a greater proportion of A grades than was previously the case with these students.

The results of the study in general supported the tenets of mastery learning theory identified in the review of the literature. Achievement tended to cluster around the criterion for mastery. As time was increased more students indicated mastery of the tasks and individual differences between students and groups of students became small. Students using the mastery strategy became more efficient in

their learning as they progressed through the unit. Achievement at any point in the sequenced unit was best predicted by achievement on the previous point. Each of these results was in agreement with mastery learning theory.

The findings reported in this thesis indicated that more students can learn what is taught in the schools than is presently the case. It was concluded that mastery learning strategies using formative testing techniques and the use of time as a variable are viable ones for use in the schools.

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CHAPTER I

THE PROBLEM

I. INTRODUCTION

> That "individual differences" exist between learners is indisputable. What is disputable is that these variations must play a role in student learning and must be reflected in our learning standards and achievement criteria. The fact that they do play a role in student learning and are reflected in the standards is due to present policies and practices rather than the necessities of the case (Bloom 1971b).

During the present century many studies have been undertaken and many programs devised which have attempted to meet the individual needs of students in our schools. Recent efforts have included extensive research on such concepts as programmed instruction, nongradedness, team teaching, and computer-assisted instruction together with the efforts of individual teachers, schools, and school systems to develop their own methods of individualized instruction. Despite this concentration of attention on the individual student, the schools of the seventies are constantly under heavy criticism from the public for providing successful and rewarding experiences for only about one-third of our learners.

Bloom (1968) proposed a mastery approach to student learning which he claimed can provide almost all students with the successful and rewarding learning experiences now attained by only a few. Using this mastery learning approach in a group-based classroom, Bloom (1973a,

1973b) claimed all or almost all students can master what is taught in our schools, students can learn more material in less time, and they will show greater interest in and attitude toward the subject learned. This report together with a companion report prepared by James Jeffrey presents the results of a project in which a mastery learning strategy was developed and carried out in the schools to examine the claims stated above.

II. STATEMENT OF THE PROBLEM

The main purpose of this study was to develop a mastery learning strategy in ninth grade algebra and to investigate the effects of the strategy on students and their learning. With respect to this purpose the following questions were asked.

1. Do the groups under the mastery and nonmastery treatments devised for this study differ with respect to the proportion of students who attain mastery on:

   (a) the basic learning task objectives?

   (b) the subunit objectives?

   (c) the unit objectives?

2. Are there differences in achievement between students under the mastery treatment and students under the nonmastery treatment on:

   (a) the basic learning tasks?

   (b) the subunits?

   (c) the unit?

3. Do changes occur in achievement from the subunit of instruction to the summative test, to the postsummative test?

4. Does a relationship exist between student aptitude and the attainment of mastery and if so, is this relationship constant over the series of subunits?

5. Do students involved with the mastery treatment become more efficient in their learning over the series of subunits?

A secondary purpose of the study was to compare the results obtained in this study with accepted mastery learning theory.

III. THE NEED FOR THE STUDY

The chances of success in the modern world are highly dependent on both the quantity and quality of educational experiences an individual has had together with the degree of success in these experiences. Bloom (1968) suggested that presently in our schools, one-third of our students learn a great deal of what we offer, one-third adequately learn the material, and one-third either fail or just barely pass in our system. Many of the administrators, teachers, parents, and students involved in education accept this as a way of life. If our educative system is to kindle a genuine interest in further learning and success in life, school learning must not be regarded as frustrating and even impossible by a sizeable proportion of educators and students. There exists a need to convince all those involved in education that most young people can learn almost all of what is taught in our schools and can have successful educational experiences.

Strategies of teaching must be developed which allow the majority of our students to attain success in the educational system. These strategies must incorporate procedures where the teacher can

concern himself with the management of learning. Carroll (1971) advocated the functions of a good teacher as:

(a) to specify what is to be learned,

(b) to motivate students to learn it,

(c) to consider individual differences while supplying instructional materials and administering them at a rate suitable for each pupil,

(d) to monitor student progress,

(e) to diagnose and remedy difficulties,

(f) to supply praise and encouragement for good performance,

(g) to provide opportunities for review and practice to ensure retention over lengthy periods of time.

The second need for this study then is to provide a strategy where the teacher may exhibit these functions and where most students can learn most of what is taught in our schools.

Several studies are discussed in Chapter II which report results in support of mastery learning strategies. However, there still exists a need to replicate these studies at different grade levels, over different content areas, in different environments, and by using different methodologies within the mastery learning concept. The third need of the present study is to provide a test of general results of previous research on mastery learning strategies.

To summarize, the need of the study is threefold.

1. To convince all those involved in education that most of the students can learn almost all of what is taught in our schools.

2. To provide an example of a working strategy where this learning can be shown to occur.

3. To provide a test of general results of previous research on mastery learning strategies.

IV. DEFINITION OF TERMS

Mastery: the attainment of a predetermined criterion.

Mastery Learning: learning defined in terms of mastery of specific sets of objectives.

Experience Oriented Mastery Learning: mastery learning involving a classroom setting in which the learner develops mathematical concepts by means of active participation.

Formative Evaluation: a process of diagnostic testing which provides immediate and regular feedback to the student and the teacher regarding the student's progress during the course of instruction.

Summative Evaluation: testing procedures used to assess the achievement of each student for the purpose of determining a grade.

Unit: the entire mathematical content dealt with during the course of this study.

Subunit: a subset of the unit comprising one-third of the content of the unit.

Basic Learning Task: a learning task, whose mastery is fundamental to the mastery of the subunit.

Terms Related to Carroll's Model of School Learning

Learning Task: the behaviour required to proceed from ignorance of some specified fact or concept to knowledge or understanding of it, or from incapability of performing some specified act to capability of performing it.

Aptitude: the amount of time an individual pupil will need to

learn a learning task under optimal learning conditions.

Opportunity: the time the learner is allowed for learning.

Quality of Instruction: the degree to which the presentation, explanation, and ordering of elements of a task to be learned approach the optimum for a given learner.

Ability to Understand Instruction: the ability of the learner to understand the nature of the task he is to learn and the procedures he is to follow in the learning of the task.

Perseverance: the time the learner is willing to spend actively engaged in learning.

V. DELIMITATIONS

1. The study was delimited by involving only 1 junior high school, 1 grade level, 2 teachers, and 153 students.

2. The study was delimited to the study of polynomial expressions as prescribed by the regular program of studies of the province of Alberta.

3. The study was delimited to the length of time spent in the school (4 weeks).

VI. OUTLINE OF THE THESIS

This first chapter has given a brief introduction to the problem and the context in which it will be discussed. Chapter II discusses several attempts at the development of mastery learning strategies and suggests characteristics of such strategies together with possible elements of a mastery learning theory. Chapter III describes the development of the materials used in the study together

with the rationale for their development. The design of the experiment, its procedures, and the methods used to analyse the data are discussed in Chapter IV. Chapter V reports the results, interpretations, and conclusions of the study. Chapter VI provides a summary of the study, discusses the results and implications of the study, and suggests possible further research areas to be studied.

CHAPTER II

THE RELATED LITERATURE

I. INTRODUCTION

The purpose of this literature review is to consider excerpts from the history of mastery learning with the intent of identifying characteristics of mastery learning strategies and the theory of learning related to them. This provides a theoretical framework within which the results of the present study are interpreted. Several attempts at mastery learning strategies developed throughout this century are discussed as is Carroll's model of school learning and Bloom's interpretation of this model to a particular mastery learning strategy. Bloom's work and related studies are examined in some detail since the present study was closely related to them. Finally, since the present study attempted to incorporate an experience learning approach to a mastery learning strategy, a brief section on activity learning is included in this review.

II. ATTEMPTS AT MASTERY LEARNING STRATEGIES

An early attempt at teaching for mastery was the Winnekta Plan initiated by Dr. Carleton Washburne in 1919. Washburne (1932) believed that a system which required every student to progress at the same rate constrained the better student and frustrated the weaker one. His plan divided the tasks of the school into two categories. The first consisted of subjects where mastery was considered to be

necessary and included reading, arithmetic, language arts, and social studies. The second consisted of group and creative activities such as drama, art, and physical education.

In those classes where mastery was essential, a years work was defined as; "what the slowest, normal, diligent child could accomplish in one year". The child who learned faster was thus able to learn at his own rate and could accomplish much more than this minimum in each year. Sequences of instructional objectives were written by the teachers in the Winnekta system and mastery was defined in terms of those objectives. Learning materials were also developed by the teachers which were usually both self-instructive and self-corrective. Commercial textbooks were seldom used.

A student worked through the materials and when he felt he had attained mastery on a set of objectives, he was given a set of exercises complete with answers. If he attained 100% of the answers correct he wrote a unit test, otherwise he was required to do another set of exercises. The student always was required to attain 100% mastery before progressing to the next set of objectives. During successive sets of objectives, reviews were essential to keep the facts fresh.

Under the leadership of Washburne, teachers in the Winnekta system accomplished the many tasks required of them to keep the system functioning, however, after his departure many modifications occurred in the system. Carroll (1970) pointed out that the system was not a complete success, probably because an adequate technology of instruction was not then available.

A second attempt at development of mastery learning techniques

was undertaken by Professor Henry Morrison at the University of Chicago in 1926. Morrison (1931) described his mastery formula as "Pre-test, teach, test the result, adapt procedure, teach, and test again to the point of actual learning". To Morrison, like Washburne before him, there was no question as to what constituted mastery: either you had acquired a piece of learning or you had not. Anything short of 100% achievement on any learning task was not mastery.

Whereas Washburne's concept of mastery was concerned with only cognitive objectives, Morrison's included affective and psychomotor objectives as well. To him life consisted of unit learnings, each of which had to be mastered before any adaptation was made. Morrison differentiated between mastery and performance by describing the application of a learning product after mastery, in its appropriate use, as performance.

Morrison recognized that if we expect each learner to absorb a given body of content in a fixed amount of time, then we should expect a wide range of performance. Thus Morrison allowed all the time a teacher required to bring almost all students to mastery on any given task. For the most part, all students participated in the teaching process, as Morrison felt that even the superior student benefited by reteaching. At other times voluntary projects and other techniques were used to enrich the experiences of the student who had attained mastery. A variety of reteaching procedures were used to assist those students who had not attained mastery. These included reteaching the same material to all, individual tutoring, restructuring activities, and redirecting student study habits.

Morrison's techniques included many that later resurfaced

with Bloom's theories. The pretest was used to orient the teacher and gave him grounds for an intelligent approach to the problem. It also established in the minds of the pupils a connection between prospective learning and present attainments. As we shall see later, Bloom's use of predetermined objectives achieved similar results. Morrison emphasized that the results of testing were purely for deciding whether the learning task had been mastered or what modification of teaching was needed if the task had not been mastered. The results were not used for a final appraisal of the student's learning. This was consistent with the ideas of Washburne and those of formative testing techniques to be discussed later.

Skinner (1954) suggested that for an individual to become competent in any field, the knowledge available in that field must be broken into a large number of very small steps and reinforcement must be contingent upon the accomplishment of each step. Skinner reasoned that with small steps the learner should give few incorrect responses, thus the frequency of correct answers, and hence positive reinforcement would be a maximum. By making the steps small and providing immediate reinforcement, a teacher could, with the aid of mechanical devices, supervise an entire class yet allow each child to progress at his own rate, completing as many problems as he could within the class period.

Skinner produced several teaching machines, all of which provided three main features: (a) small learning units, (b) immediate feedback, and (c) allowance for each learner to progress at his own rate. Early teaching machines required the student to select a correct answer from a list. Skinner developed a machine where

the learner could "write in" his own answer. This machine was modified to provide a means where the learner could correct his responses as he progressed through a unit rather than wait until he had completed it.

The characteristics of teaching machines mentioned above are all, to some extent, characteristic of mastery learning. The degree to which learning tasks need to be broken into smaller steps in a hierarchical fashion, and the time and frequency of positive reinforcement that is necessary, remain as unanswered questions. Skinner appeared at one end of the continuum, in that he advocated that the steps should be as small as possible and reinforcement immediate and often. This extreme stand, together with a very different role of the teacher, may have resulted in a reluctance to accept Skinner's concepts.

Bruner (1966) proposed that any theory of instruction has four major features. Briefly, a theory of instruction should specify:

1. the experiences which effectively implant in the individual a predisposition toward learning.

2. the ways in which a body of knowledge should be structured so that it can be most readily grasped by the learner.

3. the most effective sequences in which to present the material to be learned.

4. the nature and placing of rewards and punishment in the process of learning and teaching.

The first feature above was discussed by Bruner on the predisposition to explore alternatives. Three aspects of this exploration consist of activation, maintenance, and direction. In

mastery learning strategies activation is a result of the learner having knowledge of what he is expected to know and the assurance that he can succeed in learning the particular task. These same characteristics serve to keep the learner active in the process of learning a task. Direction is given by means of formative testing schemes which provide feedback to the student.

For the learner to master a learning task it must be structured in such a way for mastery to occur. Bruner indicated that any learning task can be presented in a form simple enough so that any particular learner can understand it. The curriculum should involve the mastery of skills, in particular in mathematics, and those skills in turn should lead to the mastery of still more powerful skills. This led Bruner to suggest a spiral curriculum where the form used in any instance must consider the mode of presentation, its economy, and its effective power, with these factors differing with respect to the ages and styles of the learners and the subject matter being considered.

Bruner recognized that the sequence in which any individual learner encounters materials affects the difficulties which he will have in achieving mastery. He suggested that the optimum sequence for most students progresses from enactive through iconic to symbolic representation of the concept. Once a learner reaches Piaget's level of formal operations he may need only symbolic representation to master a learning task. Schulman (1969) pointed out that Bruner places the emphasis in learning on process and contrasted this position to that of Gagné who suggests the emphasis be on products. It is with regard to this point that Bruner's beliefs appear to

diverge from those of mastery learning theorists. Most mastery strategies have attempted to use a hierarchical notion with respect to the structure of the content and as a result the emphasis has been on products of learning. If Bruner's notion that any learner can accommodate any task at any time as long as it is in a suitable form is interpreted to mean that a learner must possess some prerequisite behavior then the application to mastery learning would be a more direct one.

Bruner's final feature of a theory of instruction indicates the importance of the knowledge of results. He pointed out that it is important, not only to know whether one has mastered a task, but also if one is actually proceeding through the hierarchy of goals one is seeking to achieve. This information is received by the student in mastery learning conditions through the use of formative testing techniques.

Inherent in the four facets of Bruner's theory of instruction is the notion that no one approach to any facet would bring about maximum benefits for all students. Bruner stated that if a curriculum is to be effective in the classroom it must contain different ways of activating children, different ways of presenting sequences, different opportunities for some children to skip parts while others work their way through, different ways of putting things. Hence, if a mastery learning strategy is to become effective for each and every student, an important function of the teacher is to prepare for these different needs of activation, of sequences, of corrective materials, and of types of reinforcement and to diagnose and put into application those which are necessary for each student to attain mastery.

The Individually Prescribed Instruction (IPI) Project was developed at the University of Pittsburg in 1963-1964 by Glaser, Bolvin, and Lindvall under the direction of Robert Scanlon. Bolvin and Glaser (1968) listed the aims of the project to be: (a) to provide for reliably assessable individual differences among learners, (b) to develop mastery of subject matter, (c) to develop self-directed and self-initiated learners, and (d) to provide opportunities for the child to become actively involved in the learning process. To achieve these aims IPI proposed to use available information about individual differences to prescribe an appropriate educational environment for each student.

For each subject the content to be covered was stated in sequential order by behaviorally stated objectives, the objectives being grouped into small units for purposes of instruction. A student's initial contact with the program was a placement test which indicated where he was to be placed on the continuum of objectives. The student then completed a pretest on the unit of objectives indicated by the placement test. This provided him with more specific information as to where he required assistance. On the basis of these tests the teacher prepared a prescription for each student each day considering such factors as the student's general ability in the subject, the student's degree of mastery on each skill assigned, information related to the student's previous work, specific information related to the pupil's progress as he moved through the tasks, and the general learning characteristics of the student. As the student moved through the prescriptions, mastery of objectives was indicated by short curriculum embedded tests which

provided specific data on the mastery of each specific objective. At the completion of a unit a posttest was administered which was a parallel form of the pretest. A minimum score of 85% was required for progress to the next unit.

The procedures described above were carried out in self-contained classrooms where pupils were heterogeneously grouped. The curriculum taught in an IPI school was similar to that taught in any other school, however most students worked independently and moved at their own learning pace. The role of the teacher became one of a counselor, a diagnostician, and a prescriber of individualized learning experiences rather than a dispenser of content. IPI recognized the need to re-educate teachers to perform this new challenging role.

Bolvin (1969) and Lipson (1967) discussed the results of IPI programs. Bolvin pointed out that the findings indicated that IPI was in fact meeting its goals. The project had been adaptive to the individual learner as indicated by students in the same grade and same class working at a variety of levels in a variety of units, all doing different prescriptions. The time to attain mastery varied within classes and between classes. Although the achievement levels of students enrolled in IPI schools were similar to those of students from other schools, it was pointed out that the results were obtained in a system where students worked at their own ability level and were actively involved in learning. Students became more self-directed. Self-evaluation and motivation usually were not problems. Retention over the summer was reported to be high and this was attributed to the required mastery criterion. The rate of progression in the program was found not to be correlated with I.Q.

and transfer of learning was found to occur more frequently when the objectives were required to be mastered than when mastery was not required.

Divoky (1969) suggested the IPI materials were too complicated for most students and the program in general was too expensive. She claimed there was a lack of student interaction and healthy competition was eliminated from the schools.

The IPI program illustrates the effective use of several characteristics of a mastery learning strategy. The roles of the use of behavioral objectives, formative testing techniques, prescriptions, and the setting of high levels of mastery are all relevant to the present study. The program also serves to illustrate alternative uses of these concepts within a mastery learning strategy when compared with the uses in the present study. Whereas the IPI program involved almost complete individualization of instruction, the present study attempted to individualize within the framework of the same general topic for all students. The prescriptions used in IPI were made specific to the individual, whereas the correctives used in this study were similar for all students not mastering a given objective. To summarize, the IPI project illustrates the use of many of the concepts basic to the mastery strategy developed for this study but used these concepts in a more individually based manner than did this study.

The Hardisty Project, done under the direction of Mortlock and reported by Westrom (1971), Sunde (1970), and te Kampe (1970) made use of flexible grouping procedures in an attempt to individualize instruction. This project was an extension of previous work by Mortlock (1969). Basic, intermediate, and advanced objectives were

written for the content covered in these studies. The basic objectives defined the minimal behavior required to progress through the unit. The intermediate and advanced objectives involved more difficult behaviors which were not required of all students.

In the Hardisty Project the students received instruction initially (Phase I) at the intermediate level, were given a test, and on the basis of that test assigned to a basic, intermediate, or advanced group. Each group was involved with activities relevant to the particular objectives of that group (Phase II). A third phase was available to further correct difficulties or to provide enrichment activities as was necessary. In Mortlock's original study the instruction was initially given by the teacher to the entire class in Phase I and to small groups or individuals in Phase II. In the Hardisty Project instruction was given through the use of materials and handouts to the students. Independent study occupied a more central role in the Hardisty Project than in Mortlock's original study where this procedure was used only by the advanced group.

Both of these studies reported no significant differences in achievement between the experimental group and a control group when achievement was measured on a standardized test. However, the Hardisty Project reported higher achievement by the experimental group on teacher made tests. Sunde reported that in the Hardisty Project nearly two-thirds of the students attained mastery of over 80% of all intermediate objectives. It was concluded that the grouping procedures used were effective in that they allowed each student to receive instruction and achieve objectives at a level corresponding to the needs and abilities of that student. In both

Mortlock's study and the Hardisty study gains in achievement were reported from the test after Phase I to the test after Phase II. This illustrates the effective use of formative testing techniques.

As with the IPI project, the above studies illustrate several of the characteristics of a mastery learning strategy; however, they also serve to point out different ways in which characteristics are used when compared to the ways used in the present study. Again in the above studies, the use of objectives, the grouping of the students on the basis of performance on a formative-type test, and the use of correctives to remedy deficiences in the students' prior learning, all have analagous counterparts in the present study. One major difference is that in the present study, the level of mastery was the same for all students whereas in the above studies, the criterion for mastery was dependent on the group membership, and hence the mastery criterion was lower for some students than others. This was not the case with the present study.

III. CARROLL'S MODEL

Traditionally, educators have treated measures of student aptitude as indicators of how much material a student could learn and hence they allowed achievement to act as a variable in their teaching processes and treated time as a constant for all students. Carroll (1963) disputed this view by suggesting that aptitude is a measure of the time needed to learn a task and stated that any learner can succeed in learning a given task to the extent that he spends the amount of time that he needs to learn the task. Carroll assumed that the work of the school can be separated into a series of

learning tasks, placing no restrictions on these tasks other than that they can be unequivocally defined and means found to determine if the learner has achieved them. Carroll stated that the model was not intended to apply to goals such as those having to do with attitudes and dispositions which do not lend themselves to being considered as learning tasks. He believed that the acquisition of attitudes follows a different paradigm from that involved in learning tasks. This restriction did not however eliminate higher level cognitive goals from being included.

The variables involved in Carroll's model may be summarized as follows:

1. Determinants of time needed for learning
   (a) Aptitude
   (b) Ability to understand instruction
   (c) Quality of Instruction
2. Determinants of time spent in learning
   (a) Opportunity
   (b) Perseverance

Aptitude is defined as a measure of the amount of time needed to learn a given task under optimal learning conditions. Aptitude is specific to each individual task and may depend upon other characteristics of the learner and in particular upon prior learnings relevant to the task under consideration. Cronbach (1967) supported the position that rate of learning is inconsistent from one task to another while Kim (1968) found that particular aptitudes were related to learning rates for individual tasks. For example, a test on reasoning and number facility aptitudes best predicted learning rates

for statistical concepts and operations. Aptitudes best predicted learning rates for the initial and lower level skills and these learnings were in turn predictive of learning rates of higher level skills. Bloom (1973b) suggested that general aptitude and intelligence tests are good predictors of time to achieve mastery although Yeager and Kissel (1969) found that I.Q. had little predictive power. The latter reported that the student's pretest score, the number of skills to be mastered in the unit, and the student's chronological age were better predictors of the rate of learning of a particular task.

The ability to understand instruction is a general aptitude applying to all academic subjects and was referred to as a combination of general intelligence and verbal intelligence. Carroll (1971) included listening comprehension in this category and Bloom (1971) suggested that reading comprehension is also a factor. Bloom further suggested that the factor which most influences the ability of any student to understand instruction depends upon the mode of instruction used to teach the task, thus making it necessary to consider various modes of instruction for different students.

Quality of instruction, the third factor used to determine the time needed for learning a task, is the most difficult to measure. Carroll (1971) suggested that for the quality of instruction to be optimal, the material should proceed from the simple to the complex, each basic task must be mastered before proceeding to the next, and the students must understand the objectives of the lesson. The notion that instruction should proceed from simple to complex tasks suggests a hierarchial structure of the objectives of a unit.

Gagné (1962a, 1962b, 1965), Merrill, Barton and Wood (1970), and Airasian (1969) all found that more complex objectives could not be mastered before their component parts. Other factors which affect the quality of instruction included the teacher's knowledge of the subject, the quality of instructional materials, and the teacher's ability to diagnose difficulties and prescribe cures. Carroll and Spearitt (1967) found that by providing a poor quality of instruction, high, as well as low intelligence students were affected on time to criterion, perseverance, and learning efficiency.

Usually there is considerable interaction between the quality of instruction and the ability to understand instruction. Bloom (1968) stated that the student with high ability to understand instruction probably learns under less than optimal quality of instruction with few effects, however the student with low ability to understand instruction has considerable difficulty under less than optimal conditions. Again this illustrates the need to vary the mode of instruction for different learners and different tasks and to provide a high quality of instruction for all students.

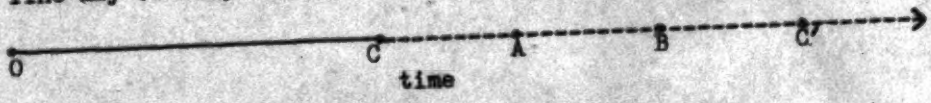
Opportunity is the time allowed for learning. Most schools offer all students the same opportunity to learn a task, hence some students have more than enough time to attain mastery, others have an adequate amount of time, while others have too little time and fail to master the learning tasks. If students are expected to attain mastery on a task, the opportunity given must be at least equal to that student's aptitude for that particular task. This becomes especially significant in a field such as mathematics where mastery of one task is a prerequisite to mastery of other tasks, and where

failure becomes certain without ample opportunity.

Perseverance, like opportunity, must be at least equal to the student's aptitude for a particular task if the student is to attain mastery of that task. Perseverance is partly a function of motivation and may depend heavily on prior successes and failures in tasks similar to the one under consideration. The other variables in the model may also have considerable effects on perseverance, in particular quality of instruction. Carroll (1971) pointed out that it is much more important to enhance perseverance than to attempt to measure or predict it. The model may be summarized by the equation:

$$\text{Degree of Learning} = f\left(\frac{\text{time actually spent}}{\text{time needed}}\right)$$

Time needed is the time required to learn under ideal conditions, increased by whatever extra time is necessary in view of less than optimal quality of instruction, and ability to understand instruction. The time actually spent is the least of the time needed, the opportunity given, and the perseverance of the student on the task. The following time line may clarify the model.



OC represents aptitude, the time in which the task may be learned under optimal conditions. When quality of instruction and ability to understand instruction are less than optimal, this time is increased to OC'. OA represents the perseverance of the student and OB the opportunity given to him to master the task. For the task to be mastered both OA and OB must be greater than or equal to OC under optimal conditions, or greater than or equal to OC' under less than optimal conditions. In the diagram the task would be mastered under

optimal conditions but not under less than optimal conditions.

IV. BLOOM'S ADAPTATION OF CARROLL'S MODEL

Bloom's (1968) strategy for mastery learning was built specifically on Carroll's model and drew from the works of Washburne, Morrison, Skinner, Bruner, and Glaser reviewed earlier. Bloom stated that if students are normally distributed with respect to aptitude for a learning task and receive exactly the same instruction (in terms of quality of instruction and opportunity), then the amount of learning will also be normally distributed. However, if the instruction and time are made appropriate to the needs and characteristics of each student, the majority of students may be expected to achieve mastery. These situations may be represented as in Figure I.

Bloom tested his strategy initially with college students. Later studies done under his supervision including those by Kim (1968), Block (1970), and Kersh (1970) applied the strategy to elementary and secondary school children. The discussion on the following pages presents an overview of the strategy and the results which were reported to be similar for various age levels and educational settings.

Bloom (1968) described his mastery learning strategy as follows:

> The approach has been to supplement regular classroom instruction by using diagnostic procedures and alternative instructional methods in such a way as to bring a large proportion of students to a predetermined standard of achievement. In this approach the goal is for most of the students to reach mastery levels of achievement within the regular term, semester, or calendar period in which the course is usually taught.

The strategy began with the notion that most students can

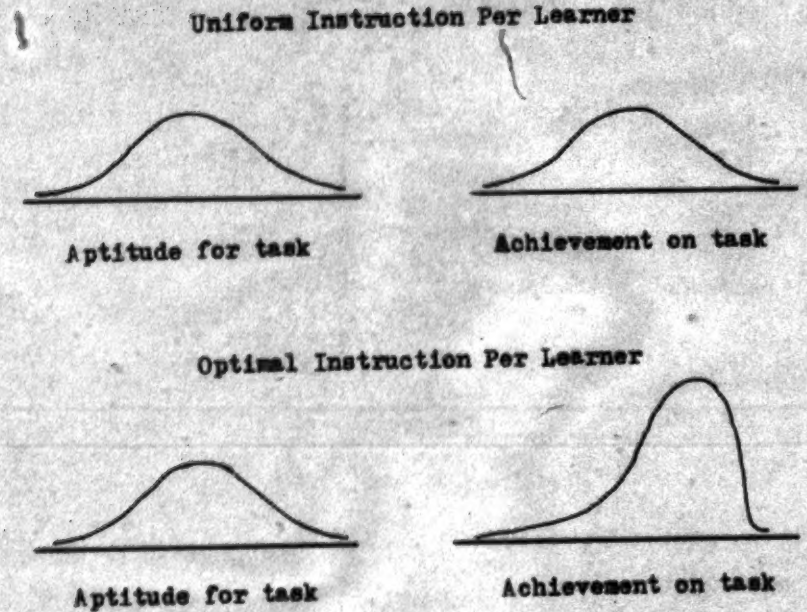


Figure I
DISTRIBUTION OF APTITUDE AND ACHIEVEMENT
FOR UNIFORM AND OPTIMAL INSTRUCTION

attain a high level of learning if instruction is approached systematically, if students are helped when and where they have difficulties, if they are given sufficient time to achieve mastery, and if there is a clear criterion of what constitutes mastery. In Bloom's studies, the material to be covered was broken into small units of 1-2 weeks work and this work was broken down into learning tasks using the ideas of Gagné (1965) to form a hierarchy. Students were made aware of exactly what these learning tasks were and what constituted mastery of the learning tasks. One criterion of mastery used by Bloom was to define mastery as the level of success required to achieve a grade of A on the same tasks during a previous year.

After the student had received regular classroom instruction on the unit, formative tests were administered which provided both

the student and the teacher with information as to where and how much assistance the student required. The formative tests were not in any way used to determine a grade for the student. Bloom suggested that students best responded to the tests when the diagnosis was accompanied by a specific prescription of alternative materials and processes that the students could use to overcome their learning difficulties. In the studies reported by Bloom, small group study sessions where students worked with each other in groups of two or three for periods of approximately one hour were used effectively to overcome difficulties. Other alternatives used were to reread previously used material, alternate textbooks, workbooks, programmed materials, or audio-visual materials. In all cases these alternatives were used outside the regular classroom period.

A summative test administered at the conclusion of the unit and after the student was given ample opportunity to remedy his difficulties was constructed on the basis of the objectives formulated prior to the group instruction. Using that test each student was given an opportunity to achieve the maximum grade available by meeting the previously established criterion for mastery. The emphasis was on encouraging a cooperative type of behavior rather than a competitive one by giving all students this opportunity.

The concepts of formative and summative evaluation, as used by Bloom, were borrowed from Scriven (1967). Scriven recognized the necessity to make the distinction between the roles and goals of evaluation and suggested that formative evaluation be used as a means of evaluating for the purposes of determining the present state of affairs and hence determining where and how improvements could be

made. Summative evaluation, on the other hand, could be used to evaluate the final product. The concept of formative evaluation is the backbone of Bloom's mastery learning strategy.

Airasian (1971), a student of Bloom, explained that formative evaluation should appear frequently during instruction and should provide immediate feedback to both teacher and learner. Although much informal formative evaluation is done by the teacher via every day classroom observation, it was suggested that short diagnostic tests may be more valuable to the student to assist him with his weaknesses. Airasian suggested three steps to be utilized in defining a hierarchy of outcomes and thus to determine a basis for formative evaluation. They were (a) identify content elements to which students have not been introduced previously, (b) define the level of cognitive functioning necessary for a student to master each new content element, and (c) specify relationships between content elements at different levels of cognitive functioning. These three steps allow the teacher to analyze the types of behavior he is expecting of his students and hence provide a map for planning and supplementing instruction. In addition, the three steps provide a blueprint for the construction of evaluative instruments, both formative and summative.

Ebel (1971) rejected the mastery learning approach discussed above and was critical of the use of criterion-referenced measurements such as formative tests. Three major limitations suggested by Ebel are that these tests tell us little of what we need to know about achievement, they are difficult to obtain on a sound basis, and they are necessary for only a small fraction of

important educational achievements. Ebel concluded that mastery learning strategies consider only the simplest of abilities and are therefore of little use. Block (1971b) responded to Ebel's criticisms by claiming that these formative instruments, though not providing us with all we need to know about achievement, do provide us with the only relevant information we do have on pupil learning. Block accused Ebel of exaggeration in his other claims and further argued that the objectives can help transform group instruction into optimal individual instruction for each learner. The learning of skills forms the bulk of each child's early school experiences and these skills are learned sequentially over several years, the learning of each skill being required of each student.

Cronbach (1963) stated, "the greatest service evaluation can perform is to identify aspects of the course where revision is necessary". Carroll (1971) suggested the two main purposes of formative testing are to find out how much pupils have learned in a restricted area of content, and to assess whether the instruction had been properly designed and conducted. Implied in these comments is the notion that the evaluation may determine that the instruction had been of no use and must be replaced by alternate methods. This has implications in a mastery learning strategy as to the type of correctives to be used.

Bloom (1973a, 1973b) discussed the results of studies completed under his direction, these results generally being consistent with the theories discussed previously. Similar results were obtained with students from elementary school through university. The evidence indicated that approximately 80% of students achieved under mastery conditions what only 20% did in nonmastery conditions. Initially in

a sequence of learning tasks students needed one to five extra units of elapsed time to attain mastery whereas later in the sequence only one to three extra units of time were necessary. Elapsed time was the record of time when the student was able to work on the task. A more valuable measure of time was time on task which gave a measure of time the student actually spent on active learning. In Bloom's studies the learner initially required one to three extra units of time and later in the sequence of tasks this reduced to one to two extra units. Bloom thus concluded that the learner not only achieved mastery, he also became more efficient in his learning. The correlation between aptitude (usually an I.Q. score) and achievement at the beginning of a sequence was quite high, in the order of 0.5-0.7, whereas later in the sequence this correlation approached 0. Formative tests given at the end of one task in the sequence tended to be predictive of achievement of the next task in the same sequence. Bloom also found that under high quality of instruction, students were more willing to persevere.

Several other studies spanning several disciplines and age groups were based on Bloom's work and lend support to it. Airasian (1972) concluded that the combination of frequent nongraded feedback and the criterion-referenced grading standards greatly increased the percentage of students who attained mastery in an introductory graduate Testing Methods course. Airasian reported that 80% of his students received an A grade whereas only 30% received an A the previous year under nonmastery conditions, but using the same criterion for an A. Airasian also concluded that substantial initial differences in a student's prior exposure to course-related skills were wiped out as

students received specific feedback about their individual learning progress.

Block (1972) defined mastery for students in a grade eight arithmetic class to be either 65, 75, 85, or 95% correct on a test. He found the higher the criterion for mastery was set, the higher were both immediate achievement and retention achievement 2 weeks later. The higher mastery levels tended to homogenize student achievement around these high scores as well as resulting in higher scores on a transfer test. Mastery groups tended to need more time on the tasks than the nonmastery groups, however there were minimal differences in time required between mastery groups with a different criteria for mastery. Some loss of interest and attitude occurred at the 95% level of mastery, hence Block concluded that in this experiment an 85% mastery level was more desireable when both affective and cognitive outcomes were desired.

Biehler (1970) found that a high percentage of students attained mastery on an undergraduate educational psychology course while using a mastery approach. Collins (1969) reported similar results on a freshman mathematics course. Other similar results were reported by Collins (1970), Kim (1967,1970), and Kersh (1970) with mathematics topics in the elementary and junior high schools. Collins (1970) also reported that a higher percentage of students attained mastery as first, objectives were added to the strategy, then diagnostic problems, and finally specific review prescriptions based on diagnostic measures.

V. ACTIVITY LEARNING

Crucial to success in the attainment of mastery is the willingness of the student to persevere. Kieren (1969) and Kieren

and Vance (1971) reviewed related literature concerning activity learning and laboratory settings for mathematics learning and reported that students enjoy an active learning approach and are willing to work actively in these situations. Low ability students in particular showed an improvement in achievement in activity learning situations, whereas other students learned at a rate comparable to those receiving instruction in a regular classroom situation.

Biggs and MacLean (1969) emphasized two things about active learning. First, a child must be allowed to do things over and over again and secondly, the practice should be enjoyable. Dienes (1960, 1964, 1967) suggested that each concept should be embodied in several different perceptual contexts and within each perceptual context, every feature should be varied that can be varied without destroying the manifestations of the concept. The works of Dienes are especially relevant at the elementary and junior high levels where most students are still working below a formal operations level.

The activity learning approach suggested that learning should be child-centered with the teacher's function to assist the child in his learning, not to divulge ideas directly to him. Rouse (1972) pointed out that in laboratory settings, we are teaching the child, not children. The time needed for any individual child to attain a concept may depend on that child's development with respect to that concept. It is the teacher's role to identify the stage and suggest appropriate activities for the student.

VI. SUMMARY

This review has presented an historical overview of several

attempts at the development of mastery learning techniques. Many of the early attempts were successful under the guidance of their originators but this success was short lived due to inadequate technology and methodology for putting the plans into widespread practice. The strategy developed by Bloom using the Carroll model of school learning overcame the difficulties of earlier attempts in that new technologies and methodologies provided correctives for the earlier deficiencies. The use of the concept of formative testing techniques was of particular importance in this regard. The research reported tended to support the Bloom strategy of mastery learning, in particular where the students were willing to persevere and take an active role in learning the tasks at hand.

Using the literature reviewed in this section, several characteristics of any mastery learning strategy were identified as were several tenets of a mastery learning theory. Characteristics of a mastery learning strategy would include the following:

1. The content area is arranged hierarchically and divided into short units of instruction.

2. Objectives are written for each learning task in the unit.

3. Mastery is defined in terms of these objectives and set at a high criterion level (greater than 80%).

4. Progress to the next unit depends on mastery of the previous unit.

5. Extensive use of formative testing techniques is used to diagnose difficulties and prescribe correctives.

6. Different types of corrective procedures are used for different students.

7. Time is used as a variable throughout the strategy.

In addition to the above characteristics, other considerations would include the size of the steps required in each unit or task, the nature and frequency of reinforcement given, and the type of thinking demanded, whether convergent or divergent.

Given a strategy which exhibits all or most of the above, the literature indicated several effects which would occur among the students, hence the following are presented as possible tenets of a mastery learning theory.

1. A high percentage of students can master the learning tasks taught in the schools.

2. The higher the criterion for mastery is set, the higher will be the achievement.

3. After a series of sequential units, student achievement will cluster around the criterion for mastery.

4. The correlation between aptitude and achievement will decrease over a series of sequential units and original differences in achievement will become less.

5. Students will become more efficient in their learning, learning more material in less time.

6. The best indicator of student achievement at any given time will be his performance on the last formative test.

7. Students will exhibit increased efforts, perseverance, and cooperation in their learning.

One purpose of this study was to develop a mastery learning strategy possessing the above characteristics and to use this strategy to examine the tenets of a mastery learning theory.

CHAPTER III

THE PREPARATION OF MATERIALS

I. INTRODUCTION

One main purpose of this study was to provide a strategy where a teacher could concern himself with the management of student learning in such a way that he would exhibit the functions of a "good teacher" as advocated by Carroll. These functions were reported on page 4 and included the notion that most students can learn almost all of what is taught in the schools. Bloom (1973a) stated that all mastery learning strategies begin with the idea that most students can achieve a high level of learning if instruction is approached systematically, if students are helped when and where they have learning difficulties, if they are given sufficient time to achieve mastery, and if there is a clear criterion of what constitutes mastery. Block (1971) suggested that any mastery learning strategy would include the following characteristics:

1. Mastery defined in terms of the particular educational objectives each student was expected to master.
2. Instruction organized into well defined learning units.
3. Mastery of each unit required before proceeding to the succeeding unit.
4. Ungraded diagnostic tests administered at the completion

of each unit to provide feedback on the adequacy of student learning.

5. Original instruction supplemented with correctives on the basis of the diagnostic tests.

6. Time used as a variable in the individualizing of instruction.

The materials described in this chapter were prepared with utmost consideration to these ideas of Carroll, Bloom, and Block as well as to achieving the purpose as previously stated.

II. SELECTION OF CONTENT AREA

Block (1971c) suggested that mastery learning techniques which have produced the best results have worked with content possessing some or all of the following characteristics:

1. The content required minimal prior learning or prior learning which most learners already possessed.

2. The content was sequentially learned.

3. The content area was closed and emphasized convergent rather than divergent thinking.

In addition to these requirements the researchers required that the content area for the present study be one from mathematics which could easily be adapted to experience oriented methods of instruction. The topic of finding the products of algebraic polynomials together with the simple factoring of these polynomials was considered by the researchers to possess all of the above characteristics.

Appendix I presents an outline of Unit III of the grade nine mathematics program at Sir George Simpson Junior High School.

Section C, subsections 3 and 4 constitute the content used in this study.

On observing this outline, it can be seen that the prior learnings necessary to the content selected for this study were minimal and of such a nature that most students should have mastered them. Also, the content which followed the selected content was for the most part an application of the selected content. The concepts in elementary algebra are sequentially learned to a high degree and since much emphasis is placed on the development of these basic algebraic skills, the area tended to emphasize convergent thinking. The area was also easily adaptable to an experience oriented approach to learning and instruction.

III. THE MATERIALS

Preparation of the Objectives

The only restriction placed on the basic learning tasks defined in this study was that each task could be unequivocally defined and means found to determine if the learner had achieved the task. Mager (1962) has outlined a method for preparing instructional objectives which satisfies the above restriction. An instructional objective was defined as a statement of educational content which communicates what the learner will be able to do after an instructional sequence, the conditions under which he must do it, and the way mastery of that objective will be determined. As Westrom (1971) has pointed out, direct application of this method often gives rise to such unwieldly statements that junior high students cannot be expected to understand them.

Westrom devised a more practical form of stating the objectives which still relied on Mager's principles but could be more easily understood by the students. It is that form of stating the objectives that was used in this study. The form consisted of presenting each basic learning task as one objective consisting of a simple statement of educational intent followed by two sample test questions together with complete solutions to those test questions. Westrom also included a statement of how the question would be marked, however in the present study this was unnecessary since each answer would be graded only as correct or incorrect.

The total content selected for the unit of study was subdivided into three subunits of instruction. An attempt was made, based mainly on the past experiences of the researchers, to make each subunit equal in both difficulty and length and one which could be mastered by most of the students within six class periods of instruction. Each class period was approximately 40 minutes in length. Each subunit of instruction was further divided into ten basic learning tasks, each learning task again being more or less equal in difficulty and length. The tasks in both the subunits and unit were arranged in such a way that each task depended on mastery of one or more of the previous tasks. An objective, as described above, was written for each basic learning task. These objectives can be found in Appendix II.

As will be seen in the following sections, these statements of objectives were used as a basis for all other materials prepared for and used in this study.

Preparation of Worksheets

The purpose of the worksheets was to provide the students with some practice questions on each basic learning task. This was necessary since the only other exposure to the task given to the student prior to the first formative test was the initial instruction.

One worksheet was prepared for each subunit of instruction, with each worksheet consisting of two questions on each task in that subunit. The items were presented in the same order as the corresponding objectives and were parallel forms of the sample questions on the objective sheets. Correct answers were provided for the students so that they could check their own work. Since the student could use the sample questions on his objective sheets as models for the corresponding questions on the worksheets, and since answers were provided, most students could answer the worksheets without further teacher assistance. Samples of the worksheets are provided in Appendix III.

Preparation of Formative Tests

Bloom (1973a) stated that the success or failure of mastery learning work is clearly related to the degree of efficiency of the formative tests in pinpointing the learning needs of each student. Block (1971c) and Airasian (1971) both have identified the purpose of formative evaluation to be one of guidance of the teaching-learning process in which immediate and continuous information regarding a student's progress during instruction is provided. Hence the formative tests became an integral part of the instructional process.

Airasian presented a three-step method of analyzing short units of learning with the aim of identifying not only the objectives

to be learned, but also the relationships between objectives. This method provided a basis for the preparation of the formative tests. These three steps are summarized as follows:

1. Identify content elements which have not been introduced to students in prior lessons.

2. Define the level of cognitive functioning necessary for a student to master each new content element.

3. Specify relationships between content elements at different levels of cognitive functioning which indicate which content elements at simpler levels are prerequisite to learning content at more complex levels.

In the framework of this study these three steps occurred in the selection of the tasks and writing of the objectives discussed earlier. The basic learning tasks for which the objectives were written constituted the new content elements. The sample problems defined the level of cognitive functioning, recalling that the content selected was treated in a convergent fashion. The relationships between content elements were also implied in the sample questions and in their solutions, as well as in the order in which they appeared in the subunit and the unit. Hence, the objectives as written served to provide a blueprint for the construction of the formative tests.

Four formative tests were prepared for each subunit with the tests for a given subunit being parallel in nature. The first formative test was designed to be administered following the initial instruction of the subunit and the succeeding ones, labeled A, B, and C to follow the use of experiences A, B, and C respectively. These

experiences will be described in the next section.

Each test contained two items on each objective in that subunit. The items were parallel forms of the corresponding examples on the objective sheets, and appeared in the same order as the objectives appeared on the objective sheets. Blanks were left following each question to allow the student sufficient space to exhibit his answer. No space was left for scratch work since only the answers were marked. To the right of each successive pair of blanks was the label of a particular experience. Hence, if a student had either question on a particular objective incorrect, the label of an experience which would assist the student in overcoming his difficulty was circled thus directing the student to that experience. These directives appeared on all formative tests except the C ones. No experiences followed the C formative tests.

The design of the formative tests explained above provided for quick and easy marking of the student responses, and thus allowed for immediate feedback in most cases. Also, the presence of the experience labels provided the student with a clear indication of which objectives he had mastered and on which ones he needed to do more work. Each item was marked only correct or incorrect and no total score was given on the formative tests. The tests were constructed so that the majority of students could complete them in less than 20 minutes. This allowed a maximum amount of time for students to correct their difficulties. These tests were not used for the purposes of obtaining marks for grading purposes when used in the mastery treatment. As mentioned previously they were for purely diagnostic purposes and were an integral part of the teaching-learning

process.

Samples of formative tests can be found in Appendix III.

Preparation of Experiences

The formative tests described in the previous section had as their main purpose to diagnose student difficulties and also to assist with the prescription of learning correctives for those difficulties. Block (1971b) has stated that the sole function of the correctives in a mastery learning strategy is to provide each student with the instructional cues and/or the active participation and practice and/or the amount and type of reinforcements he requires to complete his unit learning. The experiences described in this section were designed to serve this function and in particular, attempted to emphasize active participation by each student.

Block (1971c) described several ways of providing correctives including small group problem sessions, individual tutoring, alternative textbooks, games and puzzles, audio-visual methods, workbooks and programmed instruction, and reteaching. He emphasized that these correctives were intended to supplement and not to replace the original instruction and further suggested that they might be viewed as crutches to be used by a student at points where his original instruction was not of optimal quality. In this study the original instruction consisted of the teaching of a complete subunit within one to one and one-half class periods and the use of the worksheets described earlier. It was expected that most students would require correctives for several individual tasks.

Three experiences were prepared for each basic learning task and labeled A, B, and C. Each type of experience was constructed

following certain guidelines. The A-experiences were used first, the B-experiences second, and the C-experiences last. The number of experiences, if any, completed by the student on a given objective depended on when he exhibited mastery of the corresponding task. If he did not exhibit mastery on a task during the subunit, the number of experiences which he completed was dependent on his perseverance and the opportunity given to him.

The A-experiences were constructed to follow the first formative test. They attempted to give the student more practice on the basic learning tasks with which he had experienced difficulty. It was felt that the extra practice would rectify the difficulties of most students since the only previous work with the tasks had been on the worksheet.

The B-experiences attempted to present the student with a more detailed explanation of the steps involved in mastering the task and in most cases provided one or more extra completed examples. Most experiences of type B encouraged the student to seek assistance if he could not comprehend these extra examples. The B-experiences also served to give more practice to the student in instances where it was needed.

The C-experiences were concerned with the student who was having considerable difficulty in mastering the tasks. An attempt was made to resort to the very basic principles in explaining the tasks, giving the student encouragement that he could master the particular task. In several instances short cuts were prescribed as it was thought that might be the only method by which the student could master that particular task in the time available. The C-experiences

were also able to meet the objectives of the A and B experiences when applicable to individual students.

In all cases, the experiences were constructed with the notion that the student could complete them on his own by active participation and reference to the appropriate objective with its sample questions. The experiences often encouraged the student to seek help from a classmate or the teacher if he was having difficulty. Answers were always available in some form on the experience sheet and hence the student could determine if his solutions were correct or incorrect. An attempt was made to make the experiences interesting to the students and hence encourage active participation. This was done by using such devices as magic squares, games, anagrams, stories, dialogues, and the filling in of tables as integral parts of many experiences. These devices were used at all three levels of the experiences. The experiences were written so that most students were able to work through each experience in approximately five minutes. This time varied with the particular task and the student using the experience.

Each experience provided spaces where the student could indicate the time at which the experience was begun and the time at which the experience was completed. The purpose of obtaining this information was to determine the amount of time spent on each experience, and hence the amount of time required to attain mastery.

A complete set of experiences for subunit I is given in Appendix III.

Preparation of Review Sheets

One review sheet was prepared for each subunit of instruction

and included two questions on each objective in that subunit. The questions were not presented in the same order as the objectives. They were of a parallel form to those examples on the objective sheets. Answers were provided to the questions on the review sheet.

The review sheet was designed to be given to the student at the end of the subunit of instruction and served different purposes, depending on the student. For the student who had mastered one or more, or all of the objectives early in the subunit, the review sheet provided an opportunity for him to determine if he was still able to attain mastery criterion on those tasks. If he could not, then he could correct the deficiences. For the student who was having considerable difficulty the review sheet provided him with still more examples which he could work where the solutions were available. An example of a review sheet appears in Appendix III.

Preparation of Summative and Postsummative Tests

The purpose of the summative test was to determine theoverall achievement of the students on the 30 basic learning tasks studied in the unit of instruction. The test was administered at the conclusion of the instructional period. The postsummative test determined achievement on the same tasks and was administered 2 weeks after the instructional period had been completed. Both tests were used for the purpose of obtaining a grade on the unit of instruction.

The summative test consisted of 30 questions and included one question on each learning task. The questions were presented on the test in the same order as they appeared on the objective sheets and were similar to those examples on the objective sheets. Space was provided for the answer on the test paper, however no space was

provided for the scratch work since answers were only marked as correct or incorrect. The postsummative test was a parallel form of the summative test.

Item analyses were performed on both tests. The Pearson-product moment correlation of scores on the two tests was found to be 0.86. Table I gives the mean, variance, and Kuder-Richandson 20 Reliability Scores for both the summative and postsummative tests as determined by the scores obtained from all students involved in the study.

TABLE I

MEANS, VARIANCES, AND KUDER-RICHANDSON 20 RELIABILITY SCORES FOR SUMMATIVE AND POSTSUMMATIVE TESTS

TEST	N	MAXIMUM SCORE	MEAN	VARIANCE	K-R 20 REL.
Summative	152	30	18.2	75.9	.92
Postsummative	153	30	20.4	69.1	.92

Grades of A, B, C, D, and F were assigned corresponding to the percentages recommended in the Junior-Senior High School Handbook, 1973-74, published by the Government of Alberta. Hence a score of 24-30 received an A, that of 20-23 a B, 15-19 a C, 12-14 a D, and those below 12 a grade of F.

Copies of both the summative and postsummative tests can be found in Appendix IV.

IV. SUMMARY

This chapter has presented the reasoning behind the preparation of the materials used in this study. The materials were prepared following the guidelines suggested by Block and Airasian for the

preparation of materials for mastery learning strategies.

The materials described possessed the majority of characteristics described in Chapter II. The unit of content was divided into three subunits of material, each requiring six class periods of time. These subunits were further divided into basic learning tasks and behavioral objectives written for each task. Mastery of the tasks, subunits, and unit were all defined in terms of these objectives as explained later in this thesis. The formative testing instruments were designed so as not to affect the final grade and served both diagnostic and prescriptive functions. The three types of experiences provided for a variety of corrective procedures depending on the need of the student. The worksheets and review sheets were also considered to be a form of corrective for the students. The summative and postsummative tests were criterion-referenced tests and provided the basis for the assignment of a grade.

The movement of the student through the various materials and the use of time as a variable in the strategy will be described in detail in the next chapter in conjunction with the design of the study.

CHAPTER IV

THE EXPERIMENTAL DESIGN

The main purpose of this study was to develop a mastery learning strategy with which it could be shown that most students could learn almost all of what is taught in the schools. By developing a successful strategy all those involved in education might be further convinced that the mastery learning concept is a viable one. The previous chapter presented a description of the materials used in the study based on the rationale given there. The present chapter described the setting in which the materials were used together with the methods of usage. The questions which the study attempted to answer and the methods used to analyse the data are also stated.

I. THE SAMPLE

The sample consisted of all grade nine students enrolled in Sir George Simpson Junior High School in St. Albert. Several factors which influenced the selection of this school were: (a) a large number of students were available at one grade level in one school, (b) only two teachers taught mathematics to the grade nine students, each teaching three classes, (c) the classes were heterogeneously grouped and students had not been assigned to classes in any particular manner (a few students had been assigned to particular classes in an attempt to avoid discipline problems), (d) both teachers

were highly recommended by several sources and expressed a willingness to participate in the study, and (e) an area of content was available which was adaptable to a mastery learning strategy using an experience oriented approach.

II. ASSIGNMENT OF CLASSES TO TREATMENT

The content considered in the study was the multiplication and factoring of simple algebraic expressions as explained in more detail in the preceding chapter. This unit was divided into three subunits of instruction with each subunit representing content of equal difficulty.

The time allowed for mastery of each subunit was six class periods consisting of one class period per day for six successive school days. The classes were assigned to one of four treatments. The MMM classes received the mastery treatment for each of the three subunits of instruction. The NNN classes received the nonmastery treatment for each of the subunits. The NMM class received the nonmastery treatment for the first subunit of instruction and the mastery treatment for the second and third subunits while the NNM class received the nonmastery treatment for the first and second subunits and the mastery treatment for the third subunit.

In order to minimize the effects of quality of instruction each teacher was assigned to one MMM class and one NNN class as well as either the NMM or NNM class. It was determined by the flip of a coin that teacher X would have the NMM class, and teacher Y would have the NNM class. Each of the teachers' three classes was assigned to one of the three sequences of treatments assigned to the teacher by means of drawing slips of paper from a hat. The assignment of

treatments is summarized in Table II.

TABLE II

TREATMENTS ASSIGNED TO EACH CLASS

SUBUNIT \ CLASS	A	B	C	D	E	F
I	N	M	N	N	N	M
II	M	M	N	N	N	M
III	M	M	N	M	N	M
TEACHER	X	X	X	Y	Y	Y

III. DESCRIPTION OF MASTERY TREATMENT

The purpose of this section is to describe in detail the mechanics of the mastery treatment for a subunit of instruction.

Each subunit consisted of six class periods of learning experiences, each period being approximately 40 minutes in duration. Each class met for one period on each day except in special cases where the regular class period was missed due to other activities in the school. In these few cases it was necessary to have two class periods in one day.

On the first day of the subunit, the format of the mastery treatment was explained to the students as it is outlined in this report. Special care was taken to emphasize the role of formative testing in the program and the fact that the results of the formative testing would not in any way be used to determine a grade for the content covered. It was further emphasized that the final grade would be derived from the summative and postsummative tests and that each and every student would be judged only on how he personally achieved on these instruments. Every student, or possibly none,

could achieve an A.

The above orientation took about 10 minutes, after which the students were given the objective sheets for the particular subunit. The regular classroom teacher then presented the objectives to the students one by one by referring to the objective sheets and making use of the sample exercises on those sheets. In some cases, if the teacher felt it necessary, the sample exercises were supplemented by other exercises. In this first class period, six to eight of the ten objectives were usually presented to the students.

At the conclusion of the first class period each student was given a worksheet consisting of two questions on each of the learning tasks in the subunit. It was suggested that the student attempt each of the 20 questions before the next class period. Even though the last few objectives may not have been dealt with during the first class period, the students could have made reference to the sample questions on the objective sheets and hence should have been able to complete the worksheet.

During the second class period the remaining objectives were considered and any minor questions concerning the questions on the worksheet were answered. No formal checking of the worksheet was done. In the last half of the second period, a formative test was administered to all students in the class. Each student worked on the formative test independently and without the assistance of notes or the objective sheets. After the student had attempted each of the questions the formative test was handed in to the teacher for marking. In cases where the class time expired before a student had completed the test, the test was taken in and returned to the student the next

day for completion.

At the beginning of the third class period the formative tests were returned indicating which tasks in the subunit had not been mastered. Attached to the formative tests were A-experiences which would provide the students with experiences on the tasks on which they had not indicated mastery. An experience was provided for each task not mastered. The students worked on these experiences independently seeking assistance from the teacher, researcher, or other students when necessary.

If a student attained mastery of all ten basic learning tasks on the first formative test, or on subsequent formative tests, he was directed to an activity corner set up by the researchers where he could work on mathematical problems, play mathematical games, or engage in some other mathematical activity. These activities were chosen so as not to interact with the mathematical content considered in the experiment. The purpose of the corner was to provide activities for the students who had mastered all objectives before the completion of the six day period.

After a student had worked through the A-experiences and completed them to his own satisfaction he was given a second formative test (A). On completion of this test and as soon as possible the questions were marked correct or incorrect, and the tasks not yet mastered were indicated on the test paper. The student was then provided a B-experience for each task not yet mastered. If a student indicated mastery of a particular task on the first formative test but had the questions to that particular task incorrect on formative test A, then that task was still considered to have been mastered.

This fact was indicated to the student together with the suggestion that he might wish to work through the B-experience for that task.

After completion of the B-experiences, the student wrote another formative test (B). He then continued with C-experiences in a fashion similar to the above if he had not indicated mastery on all objectives. If he had obtained mastery on all objectives he worked in the activity corner.

A formative test C followed the C-experiences and if mastery was still not attained the student discussed his situation with the teacher. Overall then, it was possible for a student to obtain three experiences to assist him in the mastery of a given task. Whether or not he attained mastery was indicated to him by his results on the formative tests.

Since the opportunity given for the mastery of the objectives of any subunit was six class periods, several students were unable to work through all the experiences available to them. At the beginning of the sixth class period of the cycle, the students who had attained mastery of all tasks earlier were given a review sheet on the objectives in that subunit. The remaining students were given the review sheet at the end of the period. The students worked on this review sheet on their own time and no checks were done to see if the students had done the questions.

Figure II gives a flowchart of the activities completed during a subunit of instruction in a class which received the mastery treatment. It should be indicated that if the six periods allotted for the subunit ended before a student had completed mastery, the student still received the review sheet at the end of the sixth

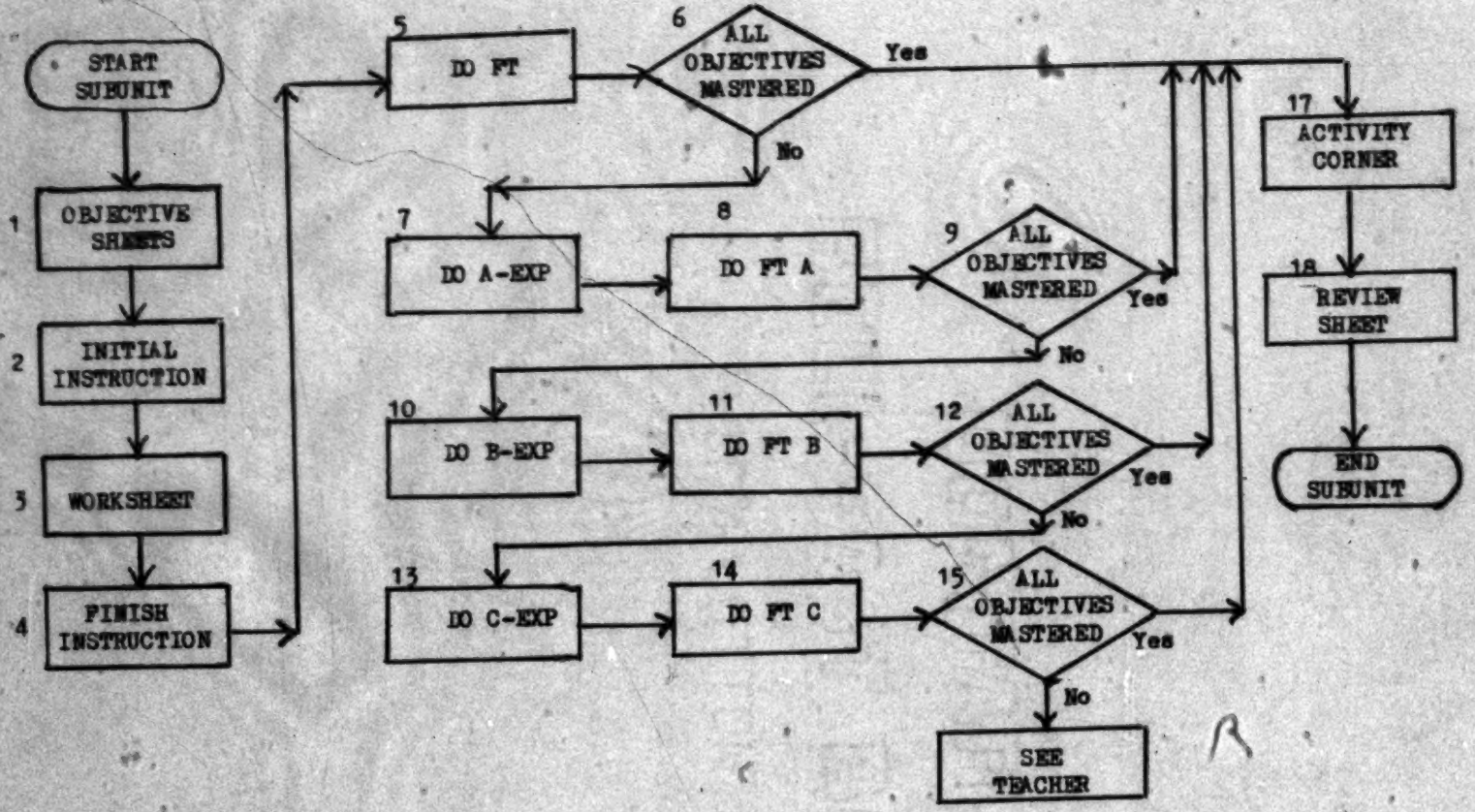


FIGURE II

FLOWCHART FOR ONE SUBUNIT OF

INSTRUCTION USING THE MASTERY TREATMENT

period and any further work done on the subunit was his own responsibility.

One or both of the researchers was present in the mastery classroom for most of the class periods. The role of the researcher in the classroom was to assist the teacher with individual help, mark the formative tests for immediate return, and record data pertinent to the study. The regular teacher was at all times responsible for the classroom.

IV. DESCRIPTION OF NONMASTERY TREATMENT

The purpose of this section is to describe in detail the mechanics of the nonmastery treatment for a subunit of instruction.

Both the length of time allotted to a subunit of instruction and the content to be covered during the subunit were identical to that involved with the corresponding subunit with the mastery treatment. The differences were with the type of instruction received by the student.

On the first day of the subunit each student who received the nonmastery treatment was given the objective sheets for that subunit. These objective sheets were identical to those given to the students who used the mastery treatment. The regular classroom teacher then presented one or more of the objectives to the students by doing several examples and using these examples as a vehicle to explain the concepts. The students were then given teacher-prepared exercise sheets on those particular objectives. The students worked on these exercise sheets both in and out of class. During the next class period these questions were corrected by such means as students working at the board, students reciting answers, or students checking answers

from a prepared answer sheet. Succeeding objectives were then taught and the process above repeated. This type of activity occurred during the first five days of the subunit and hence all ten tasks of the subunit were taught.

On the sixth day, the first portion of the class period was used for short questions about the subunit and this was followed by a test on the subunit. The test was the same as the first formative test used in the mastery treatment for that subunit with some modification occurring in the instructions. The tests were graded, the marks recorded by the teacher to be used for the purposes of obtaining a grade, and then returned to the students as soon as possible.

The approach used in the classes which received the nonmastery treatment described above was similar to the approach used regularly by their teachers. The major difference was that the students were given objective sheets indicating the tasks which they were to learn. Also, as in those classes which received the mastery treatment, one or both of the researchers attended most of the classes which received the nonmastery treatment. Their role was one of assisting students when they requested individual help. Figure III presents a flow chart representing the activities completed during one subunit of instruction by a class which received the nonmastery treatment.

V. ADMINISTRATION OF SUMMATIVE AND POSTSUMMATIVE TESTS

All students were given the summative test on the first class period following the conclusion of the third subunit of instruction. Each student was allowed as much time as he needed to complete the test. In some instances students used more than one class period of

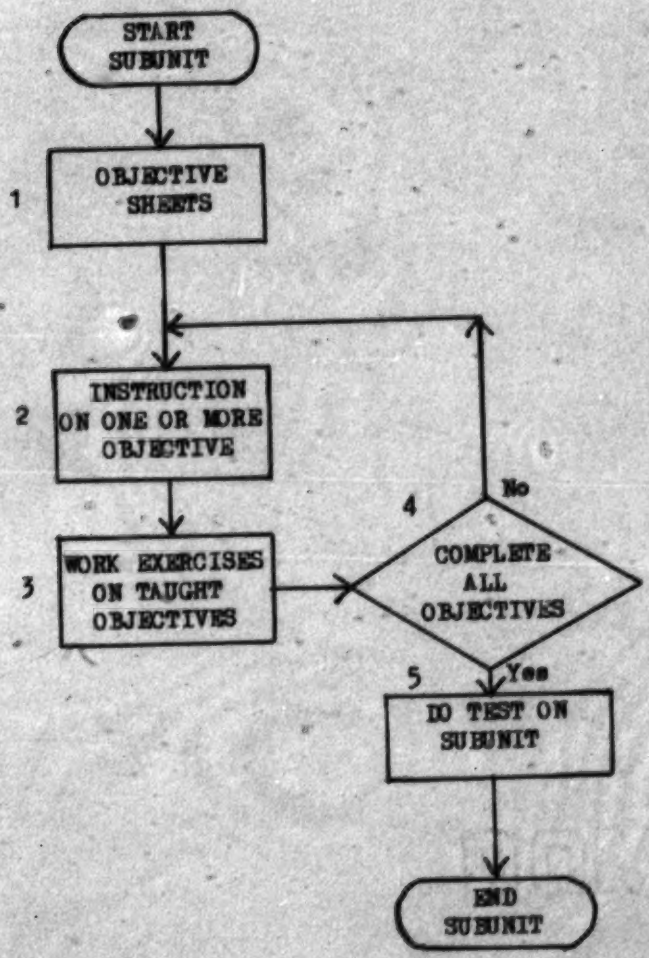


FIGURE III

FLOWCHART FOR ONE SUBUNIT OF

INSTRUCTION USING THE NONMASTERY TREATMENT

time. The tests were graded and returned to the students. Students who missed the summative test were given the test at the next possible opportunity.

During the two week period following the summative test, the first period was spent in discussion of the summative test while all remaining periods consisted of the teaching of new material. This new material made extensive use of the tasks taught during the three subunits of instruction, however no new instruction was given on those tasks. The researchers were not present in the school for this two week period and had no influence on what was taught.

Two weeks after the administration of the summative test, the postsummative test was administered in a similar fashion. The tests were graded and returned to the students. As with the summative tests the marks were used for the purpose of obtaining a grade for the student.

VI. LIMITATIONS OF THE STUDY

Because of the materials used and the design of the study, several assumptions had to be made concerning the use of those materials. The assumptions included the following:

1. Each of the subunits contained equal amounts of work.
2. Items on the formative and summative tests measured the stated basic learning task objectives.
3. Formative tests within each subunit were parallel.
4. The quality of instruction was consistent for each teacher as well as between the two teachers.
5. All subjects interpreted the items on the testing

instruments in the same way.

6. The time spent on the experiences could be measured by the number of experiences undertaken and the times recorded on the experience sheets.

7. The presence of the researchers in the classroom affected student behavior in all classes in a similar fashion.

VII. SOURCES OF DATA

Each student in the experiment was assigned an identification number which included a means of identifying the class and treatment to which that student had been assigned. Cooperative School and College Ability Test Scores (SCAT) and Sequential Tests of Educational Progress Scores (STEP) were obtained for each student from the school record. Also obtained from the school record was the grade received by each student on the last home report.

During the experiment records were kept on each student to indicate whether each response he made on a formative test was correct or incorrect as well as to indicate the total score of the test. From this information it was determined which tasks had been mastered. A record was also kept of each experience completed by a student together with his recorded time for that experience.

The total scores for each student on both the summative and postsummative tests as well as the correctness of the responses to each item were recorded. The score for each subunit on each of these tests was also noted.

Appendix V contains a record of the SCAT score, STEP score, previous achievement score, total score on the last formative test

during each subunit, and total scores on the summative and postsummative test for each student.

VIII. NULL HYPOTHESES AND STATEMENT OF ANALYSES USED

This study was concerned with five main questions together with their relation to mastery learning theory. These questions with the corresponding hypotheses which were tested to indicate how the questions should be answered are listed below. The statistical analyses used to test the hypotheses are also given.

QUESTION I

Do the groups under the mastery and nonmastery treatments devised for this study differ with respect to the proportion of students who attain mastery on (a) the basic learning tasks, (b) the subunits, and (c) the unit?

This question was answered by considering the three aspects of the mastery treatment where a mastery criterion was defined. These three aspects consisted of mastery of the basic learning tasks, mastery of the subunits, and mastery of the unit itself.

Mastery of a basic learning task was considered to be achieving two out of two items correct on a formative test administered during the subunit of instruction containing that task.

Hypothesis 1.1
There is no significant difference between the MMM group and the NMN group in the proportion of students who attain the mastery criterion on each of the basic learning tasks.

This hypothesis was repeated for each of the 30 basic learning tasks and each case tested by use of a proportions z-test in which the statistic z is defined by:

$$z = \frac{p_1 - p_2}{\sqrt{\frac{pqN}{N_1 N_2}}}$$

This statistic has a unit normal distribution and is described by Walker and Lev (1953).

Mastery of a subunit was considered to be the achievement of all ten basic learning tasks occurring in that subunit.

Hypothesis 1.2
There is no significant difference between the MMM group and the NNN group in the proportion of students who attain the mastery criterion on each of the subunits of instruction.

This hypothesis was tested three times, once for each of the subunits using the statistic z defined by the formula:

$$z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}}$$

It should be noted that for z to approximate the unit normal distribution using this statistic the product of the minimum of p_1, q_1, p_2, q_2 and the minimum of N_1 and N_2 should exceed or be equal to 5. This statistic is described by Ferguson (1971).

A second method of determining differences of attaining mastery of a subunit was to consider the mean number of tasks mastered during that subunit.

Hypothesis 1.3
There is no significant difference between the MMM group and the NNN group in the number of tasks mastered during each subunit.

Hypothesis 1.3 was tested using a t-test for testing differences in means between independent samples.

Mastery of the unit was considered to be a score of 80% on the

summative test. Any score exceeding or equal to 80% was assigned a grade of A. The following three hypotheses were tested using the same statistic as was used for hypothesis 1.2. Hypothesis 1.4 was designed to determine if there were differences in the proportion of students attaining a grade of A prior to the experiment while 1.5 and 1.6 considered if there were differences in the proportion of A's between the MMM and NNN groups on both the summative and postsummative tests.

Hypothesis 1.4
There is no significant difference between the MMM group and NNN group in the proportion of students who received a grade of A on the last home report prior to the experiment.

Hypothesis 1.5
There is no significant difference between the MMM group and NNN group in the proportion of students who attain mastery on the unit as measured on the summative test.

Hypothesis 1.6
There is no significant difference between the MMM group and NNN group in the proportion of students who attain mastery on the unit as measured on the postsummative test.

The following four hypotheses were tested to determine if either or both of the mastery and nonmastery treatments produced significant differences in the proportion of A grades attained from that previously attained by the students. If these treatments were to produce a significantly greater proportion of A grades they could be said to exhibit that quality of a mastery learning strategy. The four hypotheses were tested using the proportions z-test as before.

Hypothesis 1.7
There is no significant difference between the proportion of students in the MMM group attaining mastery on the unit as measured on the summative test and that receiving a grade of A on the last home report.

Hypothesis 1.8
There is no significant difference between the proportion of students in the MMM group attaining mastery on the unit as measured on the postsummative test and that receiving a grade of A on the last home report.

Hypothesis 1.9
There is no significant difference between the proportion of students in the NNN group attaining mastery on the unit as measured on the summative test and that receiving a grade of A on the last home report.

Hypothesis 1.10
There is no significant difference between the proportion of students in the NNN group attaining mastery on the unit as measured on the postsummative test and that receiving a grade of A on the last home report.

The first six hypotheses of this section were tested to determine if the treatments given to the MMM and NNN groups produced different results with respect to the proportion of students attaining mastery. It was expected that significant differences would result in the majority of cases and that those differences would indicate a higher proportion of students in the MMM group attaining mastery than in the NNN group. The last four hypotheses were designed to test if either or both of the mastery and nonmastery treatments produced a greater proportion of A grades than was previously attained by these groups. In order that the questions which follow have meaning it was necessary that the treatments be different and the mastery treatment exhibit some or all of the characteristics of a mastery learning strategy. The last two hypotheses above tested the possibility that the nonmastery treatment would also result in a greater proportion of students receiving a grade of A than was previously the case.

QUESTION II

Are there differences in achievement between students under the mastery treatment and students under the nonmastery treatment on (a) the basic learning tasks, (b) the subunits, and (c) the unit?

In this study there were three times at which comparisons in achievement could be made. These were during each subunit of

instruction, on the summative test, and on the postsummative test. Hypotheses were stated using one or more of these three times depending on which of the basic learning tasks, subunits, or unit was under consideration.

Hypothesis 2.1
There is no significant difference in achievement on each basic learning task between the MMM group and NNN group as measured on the summative test.

Hypothesis 2.2
There is no significant difference in achievement on each basic learning task between the MMM group and NNN group as measured on the postsummative test.

These two hypotheses tested for differences in achievement on each of the 30 basic learning tasks as measured on the summative and postsummative tests. These 60 hypotheses were tested using a proportions test as used previously to test hypothesis 1.1.

Differences in achievement on the tasks during the subunit of instruction were not considered since the testing of hypothesis 1.1 already indicated any differences in the proportion of students attaining mastery of each task. To attain mastery on a task during the subunit, the student was given two questions on that task and required to answer both questions correctly. The student could answer one question correctly and one incorrectly and hence not exhibit mastery. Therefore, the proportion of students attaining mastery of a task was not equivalent to the achievement on that task but only provided an approximation of the achievement.

Hypothesis 2.3
There is no significant difference in achievement on each subunit between the MMM group and NNN group as measured on the last formative test written during the subunit using STEP scores as a covariate.

Hypothesis 2.4
There is no significant difference in achievement on each subunit between the MMM group and NNN group as measured on the summative test using STEP scores as a covariate.

Hypothesis 2.5
There is no significant difference in achievement on each subunit between the MMM group and the NNN group as measured on the postsummative test using STEP scores as a covariate.

Each of the hypotheses 2.3, 2.4, and 2.5 was tested three times, once for each subunit of instruction. Analysis of covariance was used to test each hypothesis with STEP scores being used as the covariate, thus taking into consideration the existence of pre-arranged classes in the study.

Hypothesis 2.6
There is no significant difference in achievement on the unit between the MMM group and NNN group as measured on the summative test using STEP scores as a covariate.

Hypothesis 2.7
There is no significant difference in achievement on the unit between the MMM group and NNN group as measured on the postsummative test using STEP scores as a covariate.

These two hypotheses were tested using an analysis of covariance procedure as used to test the previous three hypotheses.

QUESTION III

Do changes occur in achievement from the subunit of instruction to the summative test, to the postsummative test?

This question was considered for the MMM group and NNN group independently. To determine the possibility that changes did occur in level of achievement, the proportion of students achieving each task during the subunit, on the summative test, and on the postsummative test were further examined. The number of proportions increasing, decreasing, and remaining the same from the subunit to the summative test, from the subunit to the postsummative test, and from the summative test to the postsummative test were determined. If this analysis revealed the majority of changes were either increasing or

decreasing, the following hypotheses would be tested to determine the significance of the changes.

Hypothesis 3.1
There is no significant difference in achievement within the MMM group on each subunit between the last formative test written during the subunit and the postsummative test.

Hypothesis 3.2
There is no significant difference in achievement within the NNN group on each subunit between the last formative test written during the subunit and the postsummative test.

Hypotheses 3.1 and 3.2 were tested for each of the three subunits of instruction using a t-test for correlated samples (Ferguson 1971). This t-test was also used to test the following two hypotheses and thus determine if changes occurred in achievement on the unit as measured on the summative and postsummative tests.

Hypothesis 3.3
There is no significant difference in achievement within the MMM group on the unit between the summative and postsummative tests.

Hypothesis 3.4
There is no significant difference in achievement within the NNN group on the unit between the summative and postsummative tests.

QUESTION IV

Does a relationship exist between aptitude and attainment of mastery and if so, is this relationship constant over the series of subunits?

Three measures of aptitude were available to this study including SCAT scores, STEP scores, and the previous grade attained by the student on his last home report. Pearson-product moment correlations were calculated between these measures and the achievement scores obtained by each student on the last formative test written during each subunit as well as on the summative and postsummative tests.

No rigorous statistical hypotheses were constructed to provide an answer to the above question. However the tables of correlation coefficients were developed to determine if trends did appear to exist.

Two other questions are discussed briefly in this section. Which of the three measures of aptitude best predict success on the subunits and unit? Do the scores on formative tests predict results on subsequent tests in later subunits? As with the main question of this section, no rigorous hypotheses were constructed, however the tables of correlation coefficients mentioned above were examined to determine if trends appeared to exist with respect to these questions.

QUESTION V

Do the students involved with the mastery treatment become more efficient in their learning over the series of subunits?

An attempt was made to answer this question by comparing the number of experiences completed during subunit III by those students who attained mastery on subunit III in the NNN, NNM, and MNM groups. In addition to comparing the number of experiences required to attain mastery, the times spent on these tasks as indicated by the student on each experience sheet were also compared. No statistical hypotheses were constructed with regard to this question.

CHAPTER V

THE RESULTS OF THE STUDY

In this chapter the results of the analysis of the data relating to the five main questions are presented with respect to the corresponding hypotheses stated in the previous chapter.

The purposes of this study were to develop a mastery learning strategy under which most students could learn most of what was taught and also to provide a test of the results of previous research concerning mastery learning strategies and theories. The first main question considered here determined if the mastery strategy devised for this study was different from the nonmastery strategy and if differences did exist, did they conform to mastery theories. The remaining four questions attempted to determine the degree of learning using the strategies, as well as to further investigate the tenets of a theory of mastery learning.

QUESTION I

Do the groups under the mastery and nonmastery treatments devised for this study differ with respect to the proportion of students who attain mastery on (a) the basic learning tasks, (b) the subunits, and (c) the unit?

The hypotheses below were tested to indicate whether there were differences in the proportion of students who attained mastery on each of the basic learning tasks, each subunit, and the unit as

measured on both the summative and postsummative tests. If there were significant differences between the two groups it could be concluded that the mastery treatment was in fact different from the nonmastery treatment. Further if the mastery group showed a significant higher proportion of students attaining mastery, then the mastery treatment would exhibit some of the tenets established in Chapter II.

Hypothesis 1.1
There is no significant difference between the MMM group and the NNN group in the proportion of students who attain the mastery criterion on each of the basic learning tasks.

The above hypothesis was tested for each basic learning task using a proportions z-test. The proportion of each group who attained mastery on each task together with the corresponding z-statistic are presented in Table III.

The MMM group was favored in 26 out of the 30 differences. The differences for items 1, 4, 6, 7, 11, 21, 23, 26, 27, 28, and 29 were significant at the .01 level and those for items 3, 9, 12, 22, 25, and 30 were significant at the .05 level. Hence, hypothesis 1.1 was rejected for those 17 items.

Hypothesis 1.2
There is no significant difference between the MMM group and the NNN group in the proportion of students who attain the mastery criterion on each of the subunits of instruction.

This hypothesis was tested using a z-statistic for proportions defined by:

$$z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}}$$

and having the restriction that the product of the minimum of p_1, q_1, p_2, and q_2 and the minimum of N_1 and N_2 must be greater than or equal

TABLE III

PROPORTIONS OF MMM AND NNN GROUPS ATTAINING MASTERY CRITERION ON EACH BASIC LEARNING TASK

ITEM	PROPORTION MASTERED IN MASTERY CLASS	PROPORTION MASTERED IN NONMASTERY CLASS	z
1	.96	.70	3.52*
2	.94	.84	1.63
3	.84	.62	2.53**
4	.82	.30	5.31*
5	.71	.68	0.06
6	.65	.28	3.70*
7	.69	.42	2.69*
8	.75	.66	0.94
9	.63	.38	2.49**
10	.57	.38	1.90
11	.63	.33	3.01*
12	.47	.27	2.13**
13	.71	.73	-0.32
14	.51	.61	-1.03
15	.49	.51	-0.20
16	.47	.41	0.69
17	.47	.53	-0.60
18	.43	.41	0.24
19	.43	.43	0.03
20	.43	.31	1.30
21	.73	.30	4.28*
22	.73	.48	2.52**
23	.86	.62	2.79*
24	.78	.76	0.29
25	.80	.58	2.44**
26	.71	.34	3.68*
27	.59	.18	4.21*
28	.57	.10	4.98*
29	.61	.30	3.11*
30	.69	.44	2.50**

* significant at .01 level
** significant at .05 level

to five. The low values of p_2 for each of the subunits as shown in Table IV would have caused this restriction to have been violated, hence for the purpose of calculating the value of z, a value of $p_2 = .10$ was used in each case.

TABLE IV

PROPORTIONS OF MMM AND NNN GROUPS ATTAINING MASTERY ON EACH SUBUNIT

SU	N_{MMM}	N_{NNN}	P_{MMM}	P_{NNN}	z
I	51	50	.37	.04	3.22*
II	51	50	.33	.06	2.84*
III	51	51	.35	.06	3.06*

* significant at .01 level

The z-statistics were significant at the .01 level and hence all three null hypotheses of the form 1.2 were rejected. It should be emphasized that the rejection of these hypotheses did not imply that the achievement of the MMM group was superior to the NNN group but only that the number of students who attained mastery on the subunit during each subunit of instruction was significantly greater.

Hypothesis 1.3
There is no significant difference between the MMM group and the NNN group in the number of tasks mastered during each subunit.

Table V presents the distribution of the number of tasks mastered in each subunit for both the MMM and NNN groups. The means and standard deviations for each group on each subunit are also given together with the t-statistic comparing the differences in the means for each subunit.

TABLE V

DISTRIBUTION OF NUMBERS OF TASKS MASTERED DURING EACH SUBUNIT BY MMM AND NNN GROUPS

NO. OF TASKS MASTERED	NUMBER OF SUBJECTS					
	SUBUNIT I		SUBUNIT II		SUBUNIT III	
	MMM	NNN	MMM	NNN	MMM	NNN
0	1	2	7	9	2	4
1	1	2	10	4	1	6
2	1	3	7	4	3	7
3	3	7	2	5	1	4
4	3	6	2	3	7	5
5	4	6	1	8	1	8
6	6	2	0	0	4	5
7	4	8	1	3	4	5
8	4	7	2	6	3	1
9	5	5	1	5	7	1
10	19	2	18	3	18	3
mean	7.41	5.44	5.00	4.46	7.08	4.16
sd	2.79	2.68	4.19	3.33	3.08	2.75
t	3.62*		.72		5.05*	

* significant at the .01 level

The t-statistics for subunits I and III were significant at the .01 level in favor of the MMM group and hence the corresponding null hypotheses were rejected. Null hypothesis 1.3 was not rejected for subunit II.

Hypothesis 1.4
There is no significant difference between the MMM group and the NNN group in the proportion of students who received a grade of A on the last home report prior to the experiment.

Hypothesis 1.5
There is no significant difference between the MMM group and the NNN group in the proportion of students who attain mastery on the unit as measured on the summative test.

Hypothesis 1.6
There is no significant difference between the MMM group and the NNN group in the proportion of students who attain mastery on the unit as measured on the postsummative test.

Table VI presents the proportion of students in both MMM and NNN groups who attained a grade of A on the summative test, postsummative test, and on the last home report prior to the experiment. The z-statistics obtained by testing the differences in proportions between the two groups are also reported.

TABLE VI

PROPORTION OF MMM AND NNN GROUPS OBTAINING A GRADE OF A ON THE SUMMATIVE TEST, POSTSUMMATIVE TEST, AND THE LAST HOME REPORT PRIOR TO THE EXPERIMENT

	MMM	NNN	z
LAST HOME RPT	.16	.08	1.26
ST	.41	.22	2.21*
PST	.51	.41	.99

* significant at .05 level

Hypothesis 1.5 was rejected since the z-statistic was significant at the .05 level. Significant differences between the two groups did not exist in the proportion of students who had attained a grade of A on the last report prior to the experiment nor did they exist in the proportion who attained mastery on the postsummative test. Hypotheses 1.4 and 1.6 were not rejected.

Hypothesis 1.7
There is no significant difference between the proportion of students in the MMM group attaining mastery on the unit as measured on the summative test and that receiving a grade of A on the last home report.

Hypothesis 1.8
There is no significant difference between the proportion of students in the MMM group attaining mastery on the unit as measured on the postsummative test and that receiving a grade of A on the last home report.

Hypothesis 1.9
There is no significant difference between the proportion of students in the NNN group attaining mastery on the unit as measured on the summative test and that receiving a grade of A on the last home report.

Hypothesis 1.10
There is no significant difference between the proportion of students in the NNN group attaining mastery on the unit as measured on the postsummative test and that receiving a grade of A on the last home report.

These hypotheses were tested using a proportions z-test. It is noted in Table VI that the proportion of students in the NNN group attaining a grade of A on the last home report was 0.08 and as before, this violated the assumptions of the test, hence it was considered that 0.10 of the students had obtained an A grade for the purposes of testing hypotheses 1.9 and 1.10. The z-statistics obtained corresponding to the four hypotheses are found in Table VII.

TABLE VII

Z-STATISTICS COMPARING PROPORTION OF A'S RECEIVED ON LAST HOME REPORT WITH PROPORTION ATTAINING MASTERY ON THE UNIT AS MEASURED ON SUMMATIVE AND POSTSUMMATIVE TESTS

	GROUP	
TEST	MMM	NNN
ST	2.80*	1.59
PST	3.72*	3.58*

* significant at .01 level

The z-statistics corresponding to hypotheses 1.7, 1.8, and 1.10 were significant at the .01 level and hence those hypotheses

were rejected. Hypothesis 1.9 was not rejected. In each of the NNN and MMM groups a greater proportion of students attained a grade of A on both the summative and postsummative tests than had previously attained a grade of A on the last home report.

Figures IV, V, and VI show respectively the distribution of grades in the MMM and NNN groups on the last home report prior to the experiment, on the summative test, and on the postsummative test.

Figure IV indicates that the distribution of grades on the last home report was similar for both groups, approximating a normal type of distribution. The distributions of grades on the summative test for the two groups were similar for the lower grades, however the MMM distributions indicated more A grades and fewer B and C grades. The MMM distribution was bimodal in nature with modes occurring at F and A. The NNN distribution was more rectangular in appearance.

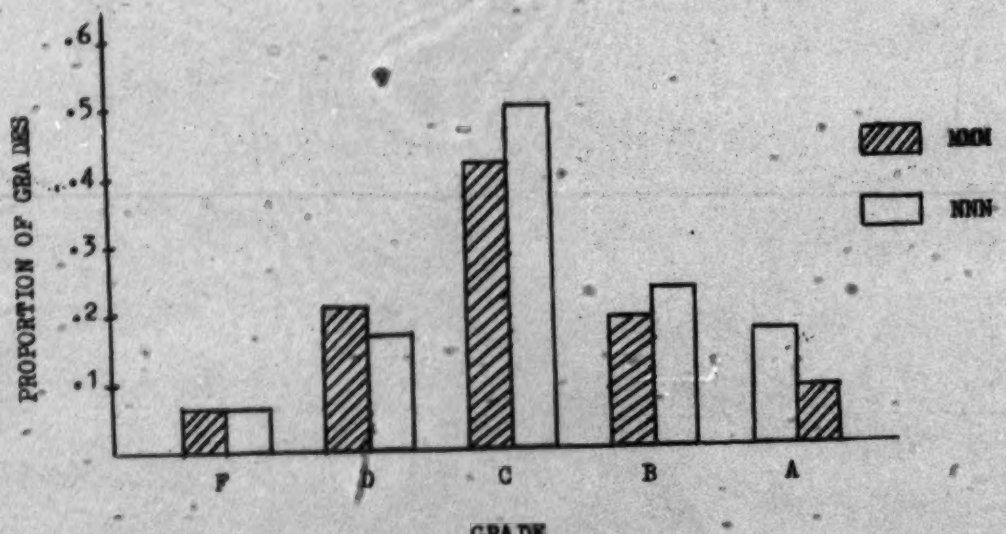


FIGURE IV
DISTRIBUTION OF GRADES ON LAST HOME REPORT PRIOR TO EXPERIMENT

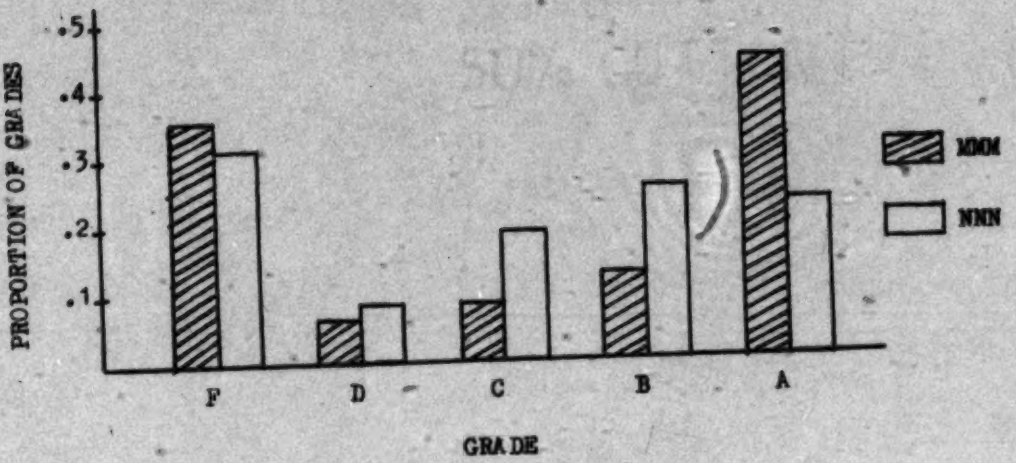


FIGURE V
DISTRIBUTION OF GRADES
ON THE SUMMATIVE TEST

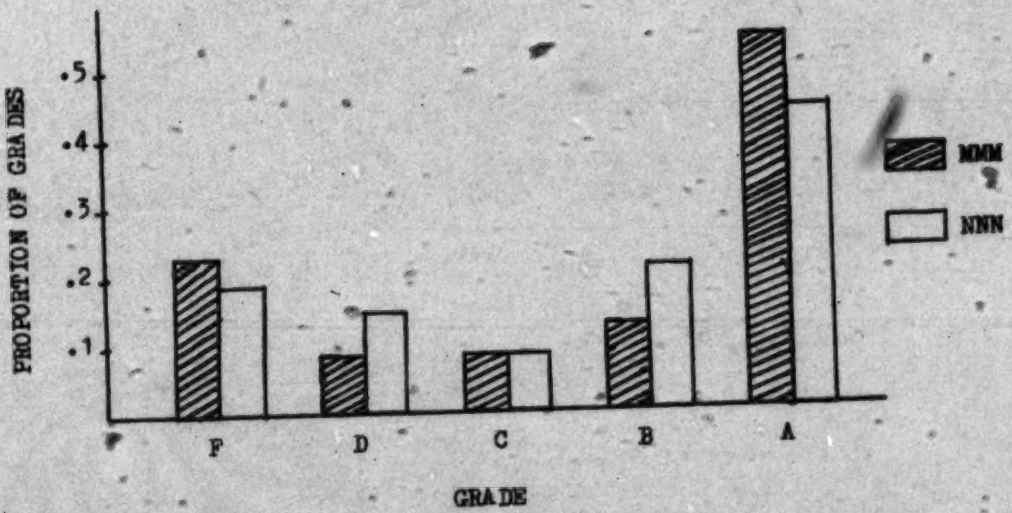


FIGURE VI
DISTRIBUTION OF GRADES
ON THE POSTSUMMATIVE TEST

The distributions of the scores on the postsummative test were very similar to each other with a modal grade of A. In both groups when the

distributions for the postsummative test were compared to those for the summative tests the peaks which represented the F grades decreased and those representing A grades increased from the summative to the postsummative tests.

The results reported in this section have indicated that the mastery and nonmastery treatments did produce differences in the proportion of students who attained mastery on the basic learning tasks, the subunits, and on the unit in the majority of cases. In those cases where the differences were not significant in favor of the MMM group, the differences favored this group except for four of the basic learning tasks. The results also indicated that the mastery treatment had some characteristics of a mastery learning strategy and suggested that the same was true of the nonmastery treatment. These possibilities will be discussed in Chapter VI of the study. Since the treatments were found to be different the remaining questions were considered.

QUESTION II

Are there differences in achievement between students under the mastery treatment and students under the nonmastery treatment on (a) the basic learning tasks, (b) the subunits, and (c) the unit?

Hypothesis 1.1 considered previously, tested whether the proportion of students in the MMM group who attained mastery on the basic learning tasks was different from that in the NNN group. That analysis also served as an indicator of differences in achievement on the tasks during the subunit. The following two hypotheses were tested to determine if there were differences in achievement on each

of the basic learning tasks on the summative and postsummative tests.

Hypothesis 2.1
There is no significant difference in achievement on each basic learning task between the MMM group and NNN group as measured on the summative test.

Hypothesis 2.2
There is no significant difference in achievement on each basic learning task between the MMM group and NNN group as measured on the postsummative test.

These hypotheses were tested for each of the 30 basic learning tasks using a proportions z-test. The proportions of each group giving correct answers together with the corresponding z-statistic are found in Table VIII and Table IX.

Hypothesis 2.1 was rejected for items 1, 8, 15, 16, 18, and 27. For items 1 and 8 the difference favored the NNN group while the remaining four significant differences favored the MMM group.

Hypothesis 2.2 was rejected for items 14 and 16. In both cases the difference favored the MMM group. It was noted that with respect to hypotheses 2.1 and 2.2, 1 difference out of 20 would be significant by chance when the .05 level of significance was used.

Table X presents a comparison of the number of proportions favoring each group on both the summative and postsummative tests. Proportions favoring a group, although not significantly, as well as those significantly favoring a group are included in the table. The table indicates that the proportion of the MMM group who gave correct responses to the basic learning task was greater than the proportion of the NNN group in 18 cases on the summative test and in 17 cases on the postsummative test, whereas the NNN group was favored in 9 and 10 cases respectively. The proportions were equal in 3 cases on each test.

TABLE VIII

PROPORTIONS OF MMM AND NNN GROUPS GIVING CORRECT RESPONSE TO ITEMS ON SUMMATIVE TEST

ITEM	PROPORTION CORRECT IN MASTERY CLASS	PROPORTION CORRECT IN NONMASTERY CLASS	z
1	.75	.90	-2.07*
2	.94	.90	0.74
3	.73	.86	-1.71
4	.73	.69	0.44
5	.77	.86	-1.27
6	.55	.61	-0.60
7	.67	.61	0.62
8	.57	.78	-2.33*
9	.49	.53	-0.40
10	.57	.57	0
11	.53	.53	0
12	.47	.45	0.20
13	.80	.86	-0.25
14	.71	.63	0.42
15	.61	.39	2.18*
16	.57	.37	1.97*
17	.61	.63	-0.20
18	.45	.26	2.07*
19	.45	.41	0.40
20	.53	.37	1.59
21	.67	.61	0.62
22	.69	.65	0.40
23	.59	.63	-0.41
24	.53	.53	0
25	.80	.71	1.15
26	.59	.53	0.60
27	.61	.39	2.18*
28	.39	.26	1.48
29	.59	.43	1.62
30	.49	.39	1.00

* significant at .05 level

TABLE IX

PROPORTIONS OF MMM AND NNN CLASSES GIVING CORRECT RESPONSE TO ITEMS ON POSTSUMMATIVE TESTS

ITEM	PROPORTION CORRECT IN MASTERY CLASSES	PROPORTION CORRECT IN NONMASTERY CLASSES	z
1	.84	.94	-1.59
2	.94	.92	0.39
3	.80	.80	0
4	.77	.78	-0.24
5	.80	.90	-1.40
6	.67	.65	0.21
7	.77	.71	0.67
8	.77	.77	0
9	.69	.65	0.42
10	.63	.61	0.20
11	.51	.67	-1.61
12	.47	.65	-1.80
13	.78	.88	-1.33
14	.82	.63	2.22*
15	.67	.61	0.62
16	.65	.43	2.19*
17	.67	.59	0.82
18	.59	.41	1.78
19	.53	.53	0
20	.57	.55	0.20
21	.65	.67	-0.21
22	.75	.65	1.08
23	.73	.82	-1.18
24	.77	.75	0.23
25	.82	.90	-1.15
26	.65	.69	-0.42
27	.67	.47	1.81
28	.49	.31	1.80
29	.57	.45	1.19
30	.67	.65	0.21

* significant at .05 level

TABLE X

PROPORTIONS FAVORING THE MMM OR NNN GROUP ON ACHIEVEMENT OF EACH BASIC LEARNING TASK

SOURCE	NUMBER OF ITEMS WHICH THE PROPORTION FAVORS MMM	NNN	SAME
ST	18	9	3
PST	17	10	3

Table XI presents the mean achievement on each subunit for each of the MMM and NNN groups as measured during the subunit, on the summative test, and on the postsummative test. The mean achievement during the subunit was based on the achievement on the test written at the end of each subunit for the NNN group, and on the last formative test written by each student during the subunit for the MMM group.

TABLE XI

ACHIEVEMENT MEANS OF EACH SUBUNIT AS MEASURED DURING THE SUBUNIT, ON THE SUMMATIVE TEST, AND ON THE POSTSUMMATIVE TEST

		MAX SCORE	X(MMM)	SD(MMM)	X(NNN)	SD(NNN)
SU I	during SU	20	14.2	5.00	13.0	4.63
	on ST	10	6.75	2.86	7.31	2.09
	on PST	10	7.67	2.65	7.73	1.96
SU II	during SU	20	10.3	7.45	10.5	6.53
	on ST	10	5.73	3.72	4.90	2.92
	on PST	10	6.24	3.39	5.94	3.10
SU III	during SU	20	14.1	5.68	10.1	5.52
	on ST	10	5.94	3.49	5.12	3.11
	on PST	10	6.76	3.05	6.35	2.98

The table indicated that during subunit I the mean favored the MMM group during the subunit but favored the NNN group on both the summative and postsummative test. During subunit II the NNN group was favored during the subunit but the MMM group was favored on the summative and postsummative tests. All three means favored the MMM group during subunit III. Hypotheses 2.3, 2.4, and 2.5 test for significant differences between those means.

Hypothesis 2.3
There is no significant difference in achievement on each subunit between the MMM group and NNN group as measured on the last formative test during the subunit using STEP scores as a covariate.

This hypothesis was tested for each subunit using analysis of covariance with STEP scores used as the covariate. It should be noted that the NNN group wrote only one test during each unit and for the purpose of the testing of this hypothesis and the succeeding ones, this test was referred to as the last formative test. Table XII summarizes the results of analysis of covariance used to test hypothesis 2.3. The adjusted means are reported in Appendix VI.

Hypothesis 2.3 was rejected at the .001 level for subunit III since the achievement of the MMM group was significantly greater than that of the NNN group. The hypothesis was not rejected for subunits I and II. The effect of the covariate was significant in each case.

Hypothesis 2.4
There is no significant difference in achievement on each subunit between the MMM group and NNN group as measured on the summative test using STEP scores as a covariate.

Hypothesis 2.5
There is no significant difference in achievement on each subunit between the MMM group and NNN group as measured on the postsummative test using STEP scores as a covariate.

TABLE XII

SUMMARY TABLE FOR ANALYSIS OF COVARIANCE RESULTS FOR ACHIEVEMENT ON EACH SUBUNIT AS MEASURED DURING THAT SUBUNIT

SOURCE	SS	DF	MS	F-RATIO
Effects (SU I)	34.8	1	34.8	1.80
Covariate	421	1	421	21.7*
Errors	1800	93	19.4	
Effects (SU II)	1.02	1	1.02	0.02
Covariate	431	1	431	9.17**
Errors	4370	93	47.0	
Effects (SU III)	371	1	371	13.0*
Covariate	421	1	421	14.8*
Errors	2650	93	28.5	

* significant at the .001 level

** significant at the .01 level

Hypotheses 2.4 and 2.5 were also tested for each subunit using analysis of covariance with STEP scores as a covariate. The results for the two hypotheses are summarized in Tables XIII and XIV respectively. The adjusted means are reported in Appendix VI.

Neither hypothesis 2.4 nor 2.5 was rejected for any of the three subunits and it was concluded that there was no significant difference in achievement on the subunits between the MMM and NNN groups as measured on both the summative and postsummative tests. The effect of the covariate was significant in each case.

Hypothesis 2.6

There is no significant difference in achievement on the unit between the MMM group and NNN group as measured on the summative test using STEP scores as a covariate.

TABLE XIII

SUMMARY TABLE FOR ANALYSIS OF COVARIANCE RESULTS FOR ACHIEVEMENT ON EACH SUBUNIT AS MEASURED ON THE SUMMATIVE TEST

SOURCE	SS	DF	MS	F-RATIO
Effects (SU I)	10.8	1	10.8	2.16
Covariate	140	1	140	27.9*
Errors	467	93	5.02	
Effects (SU II)	8.48	1	8.48	0.98
Covariate	271	1	271	31.1*
Errors	809	93	8.70	
Effects (SU III)	10.5	1	10.5	1.22
Covariate	278	1	278	32.4*
Errors	797	93	8.57	

* significant at the .001 level

TABLE XIV

SUMMARY TABLE FOR ANALYSIS OF COVARIANCE RESULTS FOR ACHIEVEMENT ON EACH SUBUNIT AS MEASURED ON THE POSTSUMMATIVE TEST

SOURCE	SS	DF	MS	F-RATIO
Effects (SU I)	1.03	1	1.03	0.21
Covariate	57.5	1	57.5	11.5*
Errors	463	93	4.80	
Effects (SU II)	1.45	1	1.45	0.17
Covariate	198	1	198	23.2*
Errors	793	93	8.53	
Effects (SU III)	1.17	1	1.17	0.18
Covariate	133	1	133	17.9*
Errors	691	93	7.43	

* significant at the .001 level

Hypothesis 2.7
There is no significant difference in achievement on the unit between the MMM group and NNN group as measured on the postsummative test using STEP scores as a covariate.

These two hypotheses were also tested using the analysis of covariance procedure with STEP scores used as a covariate. The results are reported in Table XV. The adjusted means are reported in Appendix VI.

Neither hypothesis 2.6 nor 2.7 was rejected since the F-ratios were not significant at the .05 level. The unadjusted group means were 18.4 and 17.3 for the MMM and NNN groups respectively on the summative test and 20.7 and 20.0 for the MMM and NNN groups on the postsummative test. In both cases the MMM group had the greater mean although not significantly greater. The effect of the covariate was significant in both cases.

TABLE XV

SUMMARY TABLE FOR ANALYSIS OF COVARIANCE RESULTS FOR ACHIEVEMENT ON THE UNIT AS MEASURED ON THE SUMMATIVE TEST AND POSTSUMMATIVE TEST

SOURCE	SS	DF	MS	F-RATIO
Effects (ST)	16.5	1	16.5	0.31
Covariate	2150	1	2150	40.7*
Errors	5070	96	52.8	
Effects (PST)	10.3	1	10.3	0.20
Covariate	1140	1	1140	22.1*
Errors	4960	96	51.7	

* significant at the .001 level

QUESTION III

Do changes occur in achievement from the subunit of instruction

to the summative test, to the postsummative test?

Tables III, VIII, and IX presented in conjunction with Questions I and II gave the proportion of students in each of the MMM and NNN groups who indicated mastery on each task during the subunit, on the summative test, and on the postsummative test. The number of proportions which increased, decreased, and remained the same from the subunit to the summative test, from the subunit to the postsummative test, and from the summative test to the postsummative test are presented in Table XVI. This data was given independently for each of the MMM and NNN groups. It was not determined if each change was significant, since it was the purpose here to determine if a trend did appear to exist.

TABLE XVI

CHANGES IN PROPORTION OF EACH GROUP ACHIEVING BASIC LEARNING TASKS FROM SUBUNIT TO SUMMATIVE TEST, FROM SUBUNIT TO POSTSUMMATIVE TEST, AND FROM SUMMATIVE TO POSTSUMMATIVE TEST

	INCREASED PROPORTION	DECREASED PROPORTION	CONSTANT PROPORTION
MMM SU TO ST	10	16	4
SU TO PST	17	11	2
ST TO PST	24	4	2
NNN SU TO ST	24	6	0
SU TO PST	29	1	0
ST TO PST	25	3	2

The data presented in Table XVI indicated that achievement increased on a majority of the tasks from the subunit to the summative test, to the postsummative test. An exception to this generalization

was in the MMM group from the subunit to the summative test. Since changes did occur in achievement, the following hypotheses were tested to determine the significance of those changes.

Hypothesis 3.1
There is no significant difference in achievement within the MMM group on each subunit between the last formative test written during the subunit and the postsummative test.

Hypothesis 3.2
There is no significant difference in achievement within the NNN group on each subunit between the last formative test written during the subunit and the postsummative test.

A t-test for correlated samples was used to test each of these hypotheses for each subunit of instruction. The means, standard deviations, correlations, and t-ratios for differences in achievement on each subunit within each group as measured during the subunit and on the postsummative test are presented in Table XVII. For the purpose of computing the values of t, the means and standard deviations on the postsummative test reported in the table were increased by a factor of two.

Hypothesis 3.1 was rejected for subunits I and II since the t-ratios were significant at the .01 level. Hence, a significant gain in achievement on subunits I and II occurred within the MMM group from the subunit to the postsummative test. The hypothesis was not rejected for subunit III where the achievement was less on the postsummative test than during the subunit.

Hypothesis 3.2 was rejected for all three subunits with the level of achievement being greater on the postsummative test in all three cases. The levels of significance were .01 for subunits I and III and .05 for subunit II.

Appendix VI presents tables indicating differences in

TABLE XVII

DIFFERENCES IN ACHIEVEMENT ON EACH SUBUNIT BETWEEN THE LAST FORMATIVE TEST DURING THE SUBUNIT AND THE POSTSUMMATIVE TEST WITHIN THE MMM AND NNN GROUPS

	N	X(SU) (max=20)	SD(SU)	X(PST) (max=10)	SD(PST)	r	t
MMM SU I	51	14.2	5.00	7.67	2.65	.67	-1.83**
SU II	51	10.3	7.45	6.24	3.39	.63	-2.59*
SU III	51	14.1	5.68	6.76	3.05	.71	0.85
NNN SU I	50	13.0	4.63	7.73	1.96	.42	-3.72*
SU II	49	10.5	6.53	5.94	3.11	.57	-1.71**
SU III	49	10.1	5.52	6.35	2.98	.80	-4.98*

* significant at .01 level
** significant at .05 level

achievement on each subunit between the last formative test during the subunit and the summative test and between the summative and postsummative tests.

Hypothesis 3.3
There is no significant difference in achievement within the MMM group on the unit between the summative and postsummative tests.

Hypothesis 3.4
There is no significant difference in achievement within the NNN group on the unit between the summative and postsummative tests.

These two hypotheses were tested using a t-test for correlated samples and the means, standard deviations, correlations, and t-ratios for both groups are reported in Table XVIII. All students who received the two treatments wrote both tests and were included in the analysis.

Both MMM and NNN groups exhibited significant gains in achievement on the unit from the summative test to the postsummative

TABLE XVIII

DIFFERENCES IN ACHIEVEMENT ON THE UNIT BETWEEN THE SUMMATIVE AND POSTSUMMATIVE TESTS WITHIN THE MMM AND NNN GROUPS

	N	X(ST)	SD(ST)	X(PST)	SD(PST)	r	t
MMM	51	18.4	9.48	20.7	8.45	.88	-3.57*
NNN	51	17.3	7.34	20.0	7.28	.80	-4.12*

* significant at .01 level

test. Hypotheses 3.3 and 3.4 were both rejected since the t-ratios were significant at the .01 level.

QUESTION IV

Does a relationship exist between aptitude and attainment of mastery and if so, is this relationship constant over the series of subunits?

Tables XIX, XX, XXI, and XXII present Pearson-product moment correlation coefficients for each of the MMM, NNN, NMM, and NNM groups respectively between the following variables; SCAT scores, STEP scores, previous grades, scores on the last formative test during subunits I, II, and III, and the scores on summative and postsummative tests. Whether or not the correlation coefficients are significant is also indicated in the tables.

The above question was considered using STEP scores, SCAT scores, and previous achievement as measures of aptitude whereas the total score attained on the last formative test during a subunit was used as an indicator of attainment of mastery on that subunit. The scores on the summative and postsummative test were used as indicators of the degree of mastery of the unit.

Table XIX indicates that the correlations between each of the

TABLE XIX

PEARSON-PRODUCT MOMENT CORRELATIONS FOR MMM GROUP BETWEEN SCAT SCORES, STEP SCORES, PREVIOUS ACHIEVEMENT, LAST FORMATIVE TEST IN EACH SUBUNIT, SUMMATIVE, AND POSTSUMMATIVE TESTS

	STEP	PREV ACH	SU I	SU II	SU III	ST	PST
SCAT	.76*	.70*	.64*	.57*	.53*	.65*	.61*
STEP		.61*	.65*	.53*	.48*	.74*	.68*
PREV ACH			.64*	.54*	.54*	.73*	.71*
SU I				.72*	.67*	.77*	.76*
SU II					.67*	.74*	.65*
SU III						.73*	.74*
ST							.88*

* significant at .001 level

TABLE XX

PEARSON-PRODUCT MOMENT CORRELATIONS FOR NNN GROUP BETWEEN SCAT SCORES, STEP SCORES, PREVIOUS ACHIEVEMENT, LAST FORMATIVE TEST IN EACH SUBUNIT, SUMMATIVE, AND POSTSUMMATIVE TESTS

	STEP	PREV ACH	SU I	SU II	SU III	ST	PST
SCAT	.69*	.60*	.13	.30***	.47*	.52*	.40**
STEP		.38**	.19	.14	.28	.33***	.16
PREV ACH			.39**	.45*	.64*	.68*	.60*
SU I				.53*	.49*	.44*	.46*
SU II					.74*	.65*	.64*
SU III						.87*	.80*
ST							.80*

* significant at .001 level
** significant at .01 level
*** significant at .05 level

TABLE XXI

PEARSON-PRODUCT MOMENT CORRELATIONS FOR NMM GROUP BETWEEN SCAT SCORES, STEP SCORES, PREVIOUS ACHIEVEMENT, LAST FORMATIVE TEST IN EACH SUBUNIT, SUMMATIVE, AND POSTSUMMATIVE TESTS

	STEP	PREV ACH	SU I	SU II	SU III	ST	PST
SCAT	.75*	.58**	.39	.62*	.43***	.46***	.49**
STEP		.33	.22	.38	.21	.24	.30
PREV ACH			.66*	.64*	.63*	.64*	.71*
SU I				.70*	.80*	.82*	.81*
SU II					.89*	.90*	.84*
SU III						.93*	.89*
ST							.88*

* significant at .001 level
** significant at .01 level
*** significant at .05 level

TABLE XXII

PEARSON-PRODUCT MOMENT CORRELATIONS FOR NNM GROUP BETWEEN SCAT SCORES, STEP SCORES, PREVIOUS ACHIEVEMENT, LAST FORMATIVE TEST IN EACH SUBUNIT, SUMMATIVE, AND POSTSUMMATIVE TESTS

	STEP	PREV ACH	SU I	SU II	SU III	ST	PST
SCAT	.75*	.68*	.26	.34	.61**	.47***	.40
STEP		.55*	.42	.08	.33	.33	.28
PREV ACH			.47***	.51***	.65*	.67*	.58**
SU I				.70*	.66*	.61**	.77*
SU II					.79*	.68*	.81*
SU III						.75*	.81*
ST							.87*

* significant at .001 level
** significant at .01 level
*** significant at .05 level

aptitude measures and each of the achievement measures were significantly different from zero since the probabilities of the coefficient being zero was less than .001 in all cases. Hence a relationship did exist within the MMM group between each of the aptitude measures and the attainment of mastery criterion on the subunits and the unit. The correlation between SCAT scores and achievement on the subunits decreased from 0.64 to 0.57 to 0.53 for subunits I, II, and III respectively. Using STEP scores as the measure of aptitude the corresponding correlations were 0.65, 0.53, and 0.48 while aptitude measured by previous achievement had correlations of 0.64, 0.54, and 0.54. Whether the decreases were significant or not was not determined. However it was noted that the coefficients decreased from subunit I to subunit II to subunit III for all three aptitude measures.

The correlations for the NNN group reported in Table XX tended to be lower than the corresponding ones for the MMM group. With previous achievement as a measure of aptitude, a significant relationship existed with each of the achievement measures in that the probability that the correlations were not significantly different from zero was less than .01. When SCAT tests were used as the measure of aptitude the correlation with achievement in subunit I had a 0.39 probability of not being significantly different from zero. The tendency for the correlations between aptitude and achievement to decrease from subunit I to subunit II to subunit III did not exist for the NNN group. The tendency was toward increased correlations with each successive subunit.

Within the MMM and NNN groups, Tables XXI and XXII suggested

results similar to that described for the NNN group above. The correlations of achievement with previous achievement had low probabilities of not being significantly different from zero whereas when SCAT and STEP were used as measures of aptitude many of the correlations had a greater probability of not being different from zero. The correlations again tended to increase as progress was made through the subunits in the NNM group, in particular when SCAT and previous achievement were used as the measure of aptitude. The correlations for the NMM group did not appear to exhibit these same characteristics.

Briefly, to summarize the above, relationships did appear to exist between the measures of aptitude and achievement described. The probabilities of the correlations not being significantly different from zero were very low for the MMM group for all measures of aptitude and for the other groups when previous achievement was the measure. Generally, SCAT and STEP scores as measures of aptitude did not give high correlations with achievement on the subunits or unit tests.

Two questions related to Question IV are mentioned but are not discussed in detail. The first of these asked which of the three measures of aptitude best predict success on the subunit and unit objectives. By observing Table XIX it was noted that the correlations between each of the three measures of aptitude with the measures of achievement for the MMM group did not vary to any extent. For the other three groups, the correlations are found in Table XX for the NNN group, Table XXI for the NMM group, and Table XXII for the NNM group. The correlations of previous achievement with the

achievement measures of the subunit and unit were consistently higher than those obtained with SCAT or STEP scores as the measure of aptitude. The tendency for these last three groups was for previous achievement to be the best predictor of achievement on the subunits and the unit followed by SCAT scores and STEP scores. To test these assertions the appropriate hypotheses could be constructed and tested using a t-test to determine the significance of the difference between two correlation coefficients for correlated samples.

The second question asked whether or not the scores on formative tests in a subunit are good predictors of scores on subsequent tests. Again by observing the tables the correlation between achievement on subunit I and on subunit II was greater in all four instances than the correlations between any of the three aptitude measures and achievement on subunit II. The same relationship was observed between subunits II and III, with the exception of the MMM class where the correlation was higher than with the aptitude measures but less than with subunit I. The same relationship also existed between achievement on the third formative test and the summative test and finally between the summative and postsummative test. The above indicates that achievement on any subunit or the unit consistently correlated higher with achievement on the previous test written in the sequence than on any of the aptitude measures. The best predictor of success then was achievement on the previous test.

QUESTION V

Do students involved with the mastery treatment become more efficient in their learning over the series of subunits?

The students in each of the NNM, NMM, and MMM groups who

attained mastery on subunit III were identified and the mean number of experiences and mean time in minutes spent on the experiences were calculated. These results are reported in Table XXIII.

TABLE XXIII

MEAN NUMBER OF EXPERIENCES AND AMOUNT OF TIME REQUIRED TO ATTAIN MASTERY OF SUBUNIT III BY STUDENTS IN THE NNM, NMM, AND MMM GROUPS

GROUP	NO IN GROUP	NO WHO ATTAINED MASTERY	MEAN NO OF EXP	SD	MEAN TIME REQ (min)	SD
NNN	24	11	3.55	3.01	24.8	19.4
NMM	27	5	4.80	4.21	26.2	17.8
MMM	51	18	3.50	2.18	16.6	12.9

The mean number of experiences in the NNM and MMM groups was 3.55 and 3.50 respectively with that in the NMM group being 4.80. The large variation between the NMM group and the other two groups may have been a result of the small number of students who attained mastery in that group.

The mean time in minutes for the MMM students to attain mastery was 16.6 minutes while that for the NNM and NMM groups was 24.8 and 26.2 minutes respectively. This suggested that the MMM group may have been more efficient in their learning than the other groups.

CHAPTER VI

CONCLUSIONS AND IMPLICATIONS

Much emphasis on meeting the needs of individual students has been placed on the teaching techniques used in our schools over the past decade. This study, while recognizing that these individual differences do exist, attempted to show that they need not be the factor determining how much the student learns or what his grade will be. A treatment was developed under which it was claimed that most of the students could master the majority of the ideas presented to them. This mastery treatment provided students with formative testing techniques where they could determine their difficulties and receive directions on how to overcome those difficulties. Opportunity was then made available to each student to take advantage of corrective procedures.

I. THE STUDY

The unit of content selected for the study was subdivided into three subunits of content, each of which was further divided into ten basic learning tasks. Behavioral objectives were written for each learning task, and three activity experiences designed for each objective. A worksheet, four formative tests, and a review sheet were also written for each subunit of instruction. A summative and postsummative test were constructed on the complete unit.

Both the mastery and nonmastery treatments devised for this

study covered the same content during each subunit. Both treatments presented sheets of objectives to all students at the beginning of the subunit.

In the mastery treatment, all objectives were presented during the first period and a half of the subunit with the students also given a worksheet on the objectives. All students then wrote a formative test which directed each student to experiences which assisted him with his difficulties. Each student then worked at his own rate through a sequence of formative tests and corrective experiences until he had attained mastery on the subunit or until opportunity had expired. Opportunity included three sets of experiences on each task and six class periods of time per subunit. Each student was given a review sheet at the conclusion of the subunit.

In the nonmastery treatment, the objectives were taught to the students by the methods regularly used by the classroom teachers. At the conclusion of each subunit, each student using the nonmastery treatment wrote a test on that subunit.

Two classes received the mastery treatment for all three subunits of instruction and two received the nonmastery treatment for all three subunits. A fifth class received the nonmastery treatment for the first subunit, and mastery treatment for the second and third subunits, while a sixth class received the nonmastery treatment for the first two subunits, and mastery treatment for the third. All classes wrote a summative test after the conclusion of the third subunit and also wrote a postsummative test 2 weeks later.

Two reports were written concerning the materials, treatments,

and design described. One report, written by J. Jeffrey, reported the effects of the treatments on student attitudes towards school, mathematics, and other subjects taught in the school. An attempt was also made to determine if the treatments affected the degree of cooperation and competition among students. This report has described the preparation and use of the materials developed for the study as well as their effect on the attainment of mastery and on achievement. The study also tested various tenets of mastery learning theory including relationships between student aptitude and achievement, and the relationships between time and achievement.

The study was undertaken at Sir George Simpson Junior High School in St. Albert. The sample consisted of all grade nine students in that school. Two teachers were involved in the study and the area of content studied by the students was algebraic polynomials.

II. DISCUSSION OF RESULTS

The results of this study were presented in detail in the previous chapter within the framework of the five questions stated in Chapter I. This discussion will examine the results as they relate to particular tenets of mastery learning theory. Before discussing the results, some comments will be made with respect to the materials used in the study and the design of the study as they influenced the results.

The study attempted to provide an example of a working mastery strategy where most of the students could learn almost all of the content presented. Materials prepared for any mastery learning strategy include statements of what is to be learned, formative

testing instruments which are both diagnostic and prescriptive, correctives which assist students in attaining mastery, and summative testing instruments based on the statements of what is expected. The materials prepared for this study were judged by the researchers to have possessed these characteristics on the basis of the guidelines under which they were prepared as well as on observation of their use in the experiment.

The two single most important characteristics of a mastery learning strategy are the effective use of the formative testing instruments and the use of time as a variable. As mentioned above the formative tests prepared were judged to be both diagnostic and prescriptive in nature and were used to advantage by many students in the experiment. A concerted effort was made by the researchers to assure the students that the formative tests would not be used for grading purposes, however many students remained skeptical of so much testing. An aim of any mastery learning strategy is to show all students that they can master most of the tasks and hence be successful in most of their educational experiences. The results of the formative tests indicated many students needed more assistance on most or all of the tasks. Many of the students who did not understand the purpose of the formative tests looked upon the results as a further indication of their inadequacies and not as an opportunity to have successful experiences. Many other students understood the role of the tests and used them as a road to more successful experiences.

Although time was used as a variable in the study it was necessary to restrict the amount of class time available for each subunit to six class periods. As will be seen as the results of the

study are discussed, this restriction resulted in a situation where some students did not have ample opportunity to take full advantage of all available materials. Students without ample opportunity were unable to attain mastery.

Many students who failed to attain mastery did not, in the opinion of the researchers, take full advantage of the available materials. Few students who required more experiences reached a point where they could work on C-experiences. As explained above, opportunity may not have been adequate for some students. Other students were unwilling to persevere the necessary length of time, either during or outside of class. Some students failed to use the experiences to full advantage by attempting the exercise portion of the experiences without giving adequate attention to the supplementary explanations and examples. Many other students used the experiences efficiently and in many of these cases were able to attain the mastery criterion.

Neither the worksheets nor review sheets were used effectively by many students. These instruments were designed for student use outside of regular class time and no formal check was done to determine whether or not students completed them. These two factors appeared to result in only the highly motivated student taking advantage of these instruments whereas other students failed to make any use of them.

Both the mastery and nonmastery treatments devised for this study exhibited properties of a mastery learning strategy as will be suggested by the discussion of results which follow. This result was expected for the mastery treatment, however, was not foreseen for

the nonmastery treatment. The nonmastery treatment included giving the students objective sheets and a test at the conclusion of each subunit. This test, although contributing to the final grade of the student and not meant to be formative, provided the students with an opportunity to diagnose their own difficulties and take some appropriate action before the summative test. The summative test was based on the objectives as for the mastery group. These modifications of the techniques usually used by the classroom teachers resulted in the nonmastery treatment having characteristics similar to those of a mastery strategy.

Mastery learning theory indicates that when a mastery learning strategy is used achievement will cluster around the criterion for mastery. Otherwise, achievement would usually be normally distributed. The distribution of grades for both the MMM and NNN groups on the last home report prior to the experiment closely resembled a normal distribution. There were few F and A grades and a majority of C grades. The distribution of grades received by these groups on the summative and postsummative tests was quite different. The distribution for the MMM group on the summative test was bimodal in that many received an A grade as expected, however many others received a failing grade of F. The NNN distribution also revealed a rather high proportion of F grades with a rectangular shaped distribution overall. On the postsummative test, the distributions for the two groups were almost identical. Both indicated a modal grade of A. A smaller peak also occurred at F but this had decreased from that of the summative test distributions.

Close observation of these distributions is essential to the discussion of the tenets of the theory which follow. Briefly, the

high proportion of A grades was evident, as was the increase in this proportion from the summative test to postsummative test for both groups. The clustering effect around the A grade was also clear although somewhat clouded by the large number of failing grades.

If the F grades were ignored in the distributions for the MMM group on both tests and for the NNN group on the postsummative test, the remaining portion of these distributions would be in almost total agreement with mastery learning theory. The clustering effect around the mastery criterion, the A grade, becomes very apparent with the distribution being highly skewed to the low grades.

Several factors may have been responsible for the high proportion of F grades. The corrective procedures used in the mastery treatment may not have been consistent with the methods by which some individuals best learn. For those students who experienced a great deal of difficulty, the C-experiences may have been a better first experience than the A-experiences which all students encountered. An assumption of the study was that all students had already mastered the necessary prerequisite behaviors. For many students who failed, this assumption may well have been violated. For those students who failed, the theory would indicate that either opportunity or perseverance was less than aptitude for the given tasks under the given conditions. It was implied earlier that many students did not make maximum use of the materials. Observation of the classes indicated that this was true of many of the failing students. One final reason may have been the nature of the content itself. The study of algebra was a relatively new endeavour for these students. Many students no doubt needed more opportunity to adapt to this new

field of study with its extensive terminology and use of variables. Both treatments resulted in too many failing grades. Whatever the reasons for this phenomenon, a much closer and more detailed look at the individual students receiving those grades is essential.

In this study mastery of each basic learning task required correct responses on two out of two items on any formative test during the subunit. Mastery of a subunit required mastery of all ten basic learning tasks in that subunit. Students using the mastery strategy could indicate mastery of a task on any one of a maximum of four formative tests. Those students using the nonmastery treatment had only one opportunity to indicate mastery of the tasks, that being the test written at the conclusion of each subunit. Mastery of a unit required a score of 80% or more on the summative test.

The results of the study indicated that a higher proportion of the MMM group than of the NNN group attained mastery on 26 out of the 30 basic learning tasks, on all three subunits, and on the unit. These differences in proportions were significant at least at the 0.05 level for all but nine of the basic learning tasks. The method of determining mastery of the basic learning tasks and the subunits may have favored the MMM group since they had a maximum of four opportunities to indicate mastery compared to one opportunity for the NNN group. The results on the unit where 41% of the MMM group attained mastery compared to 22% of the NNN group are more convincing since all groups had covered the same amount of material in the same amount of time and had written the same test. Mastery learning theory indicated that as many as 80% of students can attain mastery. Although the above results fell far short of this figure, the tenet

that many more students can attain mastery than is presently the case was strongly supported.

On the last home report 16% of the MMM group and 8% of the NNN group received a grade of A which was equivalent to a score of 80% or more. For both of these groups, it was noted that the proportion of A grades on the summative test was significantly greater. When the grades on the postsummative test were considered 51% of the MMM group and 41% of the NNN group attained mastery and received an A grade. These results indicated that both treatments exhibited mastery strategy characteristics.

As stated above mastery learning theory suggests that a higher proportion of students using a mastery strategy attain mastery than those who do not use such a strategy. A corollary of this tenet would be that achievement would also be greater.

Achievement on both the summative and postsummative tests was greater for the MMM group than for the NNN group. Both tests also indicated greater achievement for the NNN group on subunit I, and for the MMM group on subunits II and III. In all of these cases an analysis of covariance indicated the differences were not significant. The MMM group also indicated greater achievement on a majority of the basic learning tasks on each test. When achievement on each subunit was measured during the subunit, the achievement on subunit III was significantly greater for the MMM group than for the NNN group. The MMM group was also favored on subunit I but not on subunit II.

These results suggested that achievement for the MMM group

tended to be greater than for the NNN group. The distribution of grades previously discussed revealed many failing grades as well as a high proportion of A grades. The distributions made clear why the differences in achievement between the groups were small, whereas the differences in attainment of mastery were large.

The mastery treatment allowed the teacher much more individual contact with the student than did the nonmastery treatment where the contact was more between the teacher and the group. The opportunity for students to receive more individual assistance on problems unique to them may have influenced the high proportion attaining mastery and the greater achievement within the MMM group. This trait was examined in greater detail in the second report written concerning this study by J. Jeffrey.

A major tenet of any mastery learning strategy is that given enough time students will master the material. This tenet implies that the more time allowed the students to learn the tasks, the more students will attain mastery. In this study 2 weeks after the summative test was administered, the postsummative test was given to all students. During that 2 week period no additional formal instruction was given on the tasks however the tasks were applied in various situations. Both the MMM and NNN groups indicated an increased proportion of students attaining a grade of A from the summative to postsummative tests. Significant gains were also made in achievement on the unit as a whole as well as on each of the three subunits as measured on these tests.

The results indicated that the proportion of students achieving each task tended to increase from the subunit to the

summative test, and to the postsummative test. Similar trends also occurred for achievement on the subunits. This trend was especially evident for the NNN group. The achievement of the MMM group decreased from the subunit to the summative test in several instances, however increased on the postsummative test to where it was greater than that indicated during the subunit.

This evidence supported the notion that time is an important variable in learning. Time, together with the opportunity to apply the basic learning tasks, appeared to be essential to the attainment of mastery for many students. The decreases in achievement by the MMM group from the subunit to the summative test suggested that although the students could master the tasks in a limited amount of time, application of those tasks was essential if they were to retain the task for a longer period of time.

It has been noted that as time increased, achievement of both the MMM and NNN group tended to increase. Also the proportion of students who attained mastery was significantly greater in the MMM group and achievement tended to be greater in that group. The results also indicated that as time increased, in particular from the summative to the postsummative tests, the differences in proportions attaining mastery and in achievement between the two groups tended to become much less. This was in agreement with the theory which suggests that individual differences become minimal as time increases. It has been shown previously that this occurred among individuals within each group and it is now indicated that the phenomenon also occurred between the groups.

Further analysis of the summative tests with respect to

achievement on each subunit revealed that the further the subunit was into the past, the less the number of tasks on which achievement favored the MMM group. On the postsummative test, the number of tasks on which achievement favored each group on each subunit was consistent for all three subunits. These results further indicated that as time increased, any advantages in achievement previously held by the MMM group became less on each subunit. Also as previously indicated by the distributions of grade, achievement behavior on the unit also became consistent between the groups as time increased.

Mastery learning theory also states that the relationship between aptitude and achievement is high (0.5-0.7) at the beginning of a sequential unit of instruction. As progress is made through the unit the theory further states that this correlation decreases and finally approaches zero. The review of the related research did not reveal agreement as to what measure of aptitude best predicted achievement. This study examined three possibilities; SCAT scores, STEP scores, and the previous achievement of the student. Correlations between each of these measures of aptitude and achievement at various points in the instruction sequence were presented in Chapter V. Several trends emmerged from those correlations which reinforced tenets of mastery learning theory.

For the MMM group it was found that the correlations of each of the aptitude measures with achievement on each of the subunits decreased from subunit I to subunit II to subunit III. This trend was consistent with that indicated by mastery learning theory. In each case the decrease in correlations was small and the correlations between aptitude and achievement on the last subunit were relatively

high, in the order of 0.5. The theory would indicate a faster rate of decrease in the correlations. With the NNN, NNM, and NMM groups this trend toward decreasing correlations did not appear and in several instances the reverse trend appeared.

An attempt was made to determine which of the aptitude measures best correlated with achievement. In the MMM group all three measures of aptitude produced similar correlations with achievement on the subunits. In the other groups however, the record of previous achievement consistently correlated higher with achievement than did either SCAT or STEP scores. Correlations with STEP scores exhibited a low probability of being different from zero for each of the last three groups. These results suggested that previous achievement may be a better measure of aptitude than either SCAT or STEP scores.

Mastery learning theory indicates that aptitude correlates highly with achievement early in a sequenced unit of instruction, however as progress is made through the unit, achievement at any point in the sequence is best predicted by achievement on the previous portion of the sequence. In this study, achievement on any one subunit was consistently found to correlate higher with achievement on the next subunit in the sequence than did any of the measures of aptitude. This trend was found to occur from subunit I to the postsummative test. This illustrated strongly that in a sequenced unit of instruction it is most important that tasks occurring early in a sequence be mastered, if it is expected that the later tasks will also be mastered. The use of formative testing techniques throughout the sequence permitted the student to correct his errors as he progressed through the unit. He then was better prepared to indicate

that he had mastered the tasks on the summative instruments at the conclusion of the unit.

As a student is exposed to a mastery learning strategy over a period of time, mastery learning theory indicates that he becomes more efficient in his learning. He will in fact learn more in less time. In the study the MMM group received the mastery treatment for all three subunits of instruction, the NMM group for only the last two subunits, and the NNM group for only the third subunit. The time spent on the experiences needed to attain mastery on subunit III provide a basis to determine the viability of this tenet of the theory.

The results indicated that of those students attaining mastery of subunit III, the members of the MMM group required on the average approximately 30% less time than did the members of either of the NNM or NMM groups. This result strongly supports the theory. The theory would also indicate that the NMM group would require less time than the NNM group. In this study the two groups required similar amounts of time, however it should be noted that only five members of the NMM group attained mastery. The results for the NMM group may not be reliable due to this small number. As students become more familiar with a mastery treatment and accept the notion that they can attain mastery, then it seems highly probable that they can achieve more in less time; they can become more efficient in their learning.

Summary of the Discussion of Results

The previous discussion has compared the results found in this study to the tenets of a mastery learning theory. The results

were presented in a rigorous fashion in Chapter V and additional support or nonsupport of the tenets of the theory may be found there.

The results indicated that the mastery treatment prepared for this study did provide a working example of a mastery learning strategy where most students could learn most of the content offered. The nonmastery treatment including the application period between the summative and postsummative tests also exhibited many characteristics of a mastery strategy. It seems clear that significantly more students can master the material which is taught in our schools than is presently the case.

The results confirmed that the use of formative testing techniques and time as a variable in instruction are effective techniques in instruction. The more time given a student, the higher we can expect his achievement to be and the more likely he will attain mastery. This appeared to be especially evident when during the additional time, he was given opportunity to apply the tasks he was expected to master.

The following results supported corresponding tenets of the theory.

1. Achievement clustered around the mastery criterion.

2. Previous individual differences in achievement became small.

3. Aptitude correlated highly with achievement early in a sequential unit but this correlation decreased as progress was made through the unit.

4. Previous achievement was as good as or better measure of aptitude than either STEP or SCAT scores.

5. Achievement at any point in a sequential unit correlated highly with achievement on the succeeding point.

6. Students became more efficient in their learning as they progressed through a unit using the mastery treatment.

The reader of this report might wish to disagree with the interpretation of results which suggests support for mastery learning theory. Arguments providing possible explanations of the results other than the one presented in this chapter might include the following.

1. Differences may have existed between the groups prior to the study.

2. The MMM group received an individualized form of instruction, whereas the NNN group received group based instruction.

3. The mastery strategy provided students with more practice in taking tests than did the nonmastery strategy.

4. The presence of the researchers in the classroom may have biased the results in favor of the mastery strategy.

5. The instruments used to evaluate achievement may have favored the mastery strategy.

The support for the theory presented in this study has not been overwhelming for any particular tenet of the theory. However almost all the data obtained did support some tenet of the theory, and most tenets were supported to some degree. This writer is prepared to suggest that mastery learning theory is one with which every educator should acquaint himself and make use of in the classrooms of our schools.

III. IMPLICATIONS

The discussion of the results has suggested that the theory of mastery learning is a viable theory and one which offers a framework within which strategies can be developed for implementation in the schools. Essential to the success of such strategies is the acceptance of the notion that most students can learn almost all of what is taught in our schools.

A mastery learning strategy is intended to supplement rather than replace the regular classroom instruction of the teacher, however the roles of teachers and students may undergo change. Teachers will have to select or prepare instructional materials which best meet the learning needs of the students in their classrooms. This means the teacher must have extensive knowledge of each individual he teaches in order to prescribe the best possible corrective procedures. Planning and preparation of materials is a task demanding the participation and cooperation of several teachers to derive maximum benefits. Each teacher using such a strategy would need to exhibit the characteristics of a good teacher described by Carroll and outlined in Chapter I. In particular, he must take care to specify what is to be learned, to consider individual differences and to diagnose and remedy student difficulties.

The student must become an active rather than passive recipient of knowledge. He must be willing to pursue the corrective procedures indicated by the formative instruments and persevere until he has attained mastery. If the strategy is in fact a mastery strategy, the theory claims the student will assume this active role.

Although the theory has been shown to be a viable one, there

are many aspects where further research is needed. The nature of the initial instruction could be investigated with regard to the number as well as to the scope of learning tasks presented. How much instruction should be given before a formative test is given to the students? How often and at what rate are these formative tests given to the students? Observation of the classes in the present study indicated that the strategy used may have included too much formative testing in a short period of time.

In order to be able to prescribe suitable correctives to each individual student, more research needs to be done on how students possessing certain characteristics best learn mathematics. What affects the ways students learn particular content under certain conditions? What type of corrective procedures best suits the needs of what type of student?

A mastery learning strategy requires the compilation of a great amount of information on each student. His past history, his home life, and his previous knowledge of the subject all become central features in his present learning. The options available to him are also numerous; he can continue with more instruction, write formative tests, or pursue one of many possible corrective procedures. The use of the computer would no doubt become a valuable aid in the management of a mastery learning strategy.

One last recommendation for research involves the applicability of the theory to long periods of time and to several disciplines simultaneously. What are the effects when all subjects are taught using a mastery learning strategy? What are the long term effects obtained by using the strategy? These questions can only be answered

within large research projects over long periods of time.

In summary, recommendations for further research related to mastery learning theory and strategies include the following.

1. Where and how frequent should formative testing techniques be used in a strategy?

2. How do students possessing certain characteristics best learn mathematics and what types of corrective procedures will best assist these students in attaining mastery?

3. How can the computer be effectively used in mastery learning strategies?

4. What are the long term effects of mastery learning strategies?

BIBLIOGRAPHY

BIBLIOGRAPHY

Airasian, P. W. An Application of a Modified Version of John Carroll's Model of School Learning. Unpublished Master's thesis, University of Chicago, 1967.

Airasian, P. W. Formative Evaluation Instruments: A Construction and Validation of Tests To Evaluate Learning Over Short Time Periods. Unpublished doctoral dissertation, University of Chicago, 1969.

Airasian, P. W. The Role of Evaluation in Mastery Learning. Mastery Learning Theory and Practice. J. H. Block (Ed.), New York: Holt, Rinehart, & Winston, Inc., 1971, 77-88.

Airasian, P. W. An Application of a Mastery Learning Strategy. Psychology in the School. April 1972, 130-134.

Atkinson, R. C. Computer-based Instruction in Initial Reading. Proceedings of the 1967 Invitational Conference on Testing Problems. Princeton: Educational Testing Service, 1968.

Banwell, C. S., Saunders, K. D., & Tahata, D. C. Starting Points. London: Oxford University Press, 1972.

Biehler, R. F. A First Attempt at a 'Learning For Mastery' Approach. Educational Psychologist, 1970, 7(3), 7-9.

Biggs, E. E., & MacLean, J. R. Freedom to Learn. Don Mills, Ontario: Addison Wesley Ltd., 1969.

Block, J. H. The Effects of Various Levels of Performance on Selected Cognitive, Affective, and Time Variables. Unpublished doctoral dissertation, University of Chicago, 1970.

Block, J. H. Criterion-referenced Measurements: Potential. School Review, February 1971, 79, 289-297.

Block, J. H. (Ed.) Introduction to Mastery Learning: Theory and Practice. Introduction to Mastery Learning Theory and Practice. New York: Holt, Rinehart, & Winston, Inc., 1971.

Block, J. H. (Ed.) Operating Procedures For Mastery Learning. Introduction to Mastery Learning Theory and Practice. New York: Holt, Rinehart, & Winston, Inc., 1971.

Block, J. H. Student Learning and the Setting of Mastery Performance Standards. Educational Horizons, Summer 1972, 183-191.

Bloom, B. S. Learning For Mastery. Evaluation Comment, UCLA, May 1968, 1(2).

Bloom, B. S. Some Theoretical Issues Relating to Educational Evaluation. Educational Evaluation: New Roles, New Means. Ralph Tyler (Ed.), 68th Yearbook, NSSE, Chicago: University of Chicago Press, 1969.

Bloom, B. S. Mastery Learning. Mastery Learning Theory and Practice. New York: Holt, Rinehart, & Winston, Inc., 1971, 47-63.

Bloom, B. S. Innocence in Education. School Review, May 1972, 80, 1-17.

Bloom, B. S. An Introduction To Mastery Learning Theory. Paper presented at the meeting of the American Education Research Association, New Orleans, February 1973.

Bloom, B. S. Time and Learning. Paper presented to the American Psychological Association, Montreal, August 1973.

Bloom, B. S., Hastings, J. T., & Madaus, G. F. Handbook of Formative and Summative Evaluation. Toronto: McGraw-Hill, 1971.

Bolvin, J. O. Individually Prescribed Instruction--Curricular Organization and Research Findings. Individualized Curriculum and Instruction, Proceedings of the Third Invitational Conference on Elementary Education, Banff, Alberta, October 29-November 1 1969.

Bolvin, J. O., & Glaser, R. Developmental Aspects of Individually Prescribed Instruction. Audio-visual Instruction, October 1968, 13(8), 828-831.

Bruner, J. S. Toward a Theory of Instruction. Cambridge, Mass.: Harvard University Press, 1966.

Buros, O. K. (Ed.) Sixth Mental Measurements Yearbook. Highland Park, New Jersey: Gryphon Press, 1965.

Buros, O. K. (Ed.) Seventh Mental Measurements Yearbook. Highland Park, New Jersey: Gryphon Press, 1972.

Carroll, J. H. A Model of School Learning. Teachers College Record, 1963, 64, 723-733.

Carroll, J. B. Problems of Measurement Related to the Concept of Learning For Mastery. Introduction to Mastery Learning Theory and Practice. J. H. Block (Ed.), New York: Holt, Rinehart, & Winston, Inc., 1971, 29-46.

Carroll, J. B., & Spearitt, D. A Study of a Model of School Learning. Monograph No. 4. Cambridge, Mass.: Harvard University, Center of Research and Development of Educational Differences, 1967.

Collins, K. M. A Strategy For Mastery Learning in Freshman Mathematics. Unpublished Study, Purdue University, Division of Mathematical Sciences, 1969.

Collins, K. M. A Strategy For Mastery Learning in Modern Mathematics. Unpublished Study, Purdue University, Division of Mathematical Sciences, 1970.

Cronbach, L. J. Course Improvement Through Evaluation. Teacher's College Record, 1963, 64, 672-683.

Cronbach, L. J. How Can Instruction Be Adapted To Individual Differences. Learning and Individual Differences. Robert Gagné (Ed.), Columbus: Merrill Publishing Company, 1967.

Dienes, Z. P. Building Up Mathematics. London: Hutchinson Educational Ltd., 1960.

Dienes, Z. P. The Power of Mathematics. London: Hutchinson Educational Ltd., 1964.

Divoky, D. Individually Prescribed Instruction. Nations Schools, November 1969, 84(5), 44-46.

Ebel, R. L. Criterion-referenced Measurements: Limitations. School Review, February 1971, 79, 282-288.

Ferguson, G. A. Statistical Analysis in Psychology and Education. Toronto: McGraw-Hill, 1971.

Gagné, R. M. Factors in Acquiring Knowledge of a Mathematical Task. Psychological Monographs, 1962, 76.

Gagné, R. M. The Acquisition of Knowledge. Psychological Review, 1962, LIX.

Gagné, R. M. The Conditions of Learning. New York: Holt, Rinehart, & Winston, Inc., 1965.

Glaser, R. Adapting the Elementary School Curriculum to Individual Performances. Proceedings of the 1967 Invitational Conference on Testing Problems. Princton: Educational Testing Service, 1968.

Goodlad, J. I., & Anderson, R. H. The Non-graded Elementary School. New York: Harcourt Brace, 1959.

Government of Alberta. Junior-Senior High School Handbook 1973-74.

Kersh, M. E. A Strategy for Mastery Learning in Fifth Grade Arithmetic. Unpublished doctoral dissertation, University of Chicago, 1970.

Kidd, K. P., Meyers, S. S., & Cilly, D. M. The Laboratory Approach to Mathematics. Chicago: Service Research Associates, 1970.

Kieren, T. E. Activity Learning. Review of Educational Research, October 1969, 39, 509-522.

Kieren, T. E., & Vance, J. H. Laboratory settings in mathematics: What the research says to the teacher. The Arithmetic Teacher, December 1971, 18, 585-589.

Kim, H. Learning Rates, Attitudes and Achievements. Unpublished doctoral dissertation, University of Chicago, 1968.

Kim, H. A Study of the Bloom Strategies for Mastery Learning. Seoul: Korean Institute for Research in Behavioral Sciences, 1969.

Kim, H. The Mastery Learning Project in the Middle Schools. Seoul: Korean Institute for Research in Behavioral Sciences, 1969.

Krulick, S. A Mathematics Laboratory Handbook for Secondary Schools. Philadelphia: W. B. Saunders Company, 1972.

Lindvall, C. M., & Bolvin, J. O. The Role of the Teacher in Individually Prescribed Instruction. Educational Technology, February 1970, 10(2), 37-41.

Lindvall, C. M., & Cox, R. C. The Role of Evaluation in Programs for Individualized Instruction. Educational Evaluation: New Roles, New Means. Ralph Tyler (Ed.), 68th Yearbook, NSSE, Chicago: University of Chicago Press, 1969.

Lipson, J. I. Individualized Instruction in Elementary Mathematics. Research in Mathematics Education, Washington, National Council of Teachers of Mathematics, 1967, 70-79.

Mager, R. M. Preparing Instructional Objectives. Palo Alto, Calfornia: Fearon Publishers, 1962.

Merrill, M. D., Barton, K., & Wood, L. E. Specific Review in Learning a Hierarchial Imaginery Science. Journal of Educational Psychology, 1970, 61, 102-109.

Morrison, H. C. The Practice of Teaching in the Secondary School. (2nd ed.) Chicago: University of Chicago Press, 1931.

Mortlock, R. S. Provisions for Individual Differences in Eleventh Grade Mathematics Using Flexible Grouping Based on Achievement of Behavioral Objectives: An Exploratory Study. Unpublished doctoral dissertation, University of Michigan, 1969.

Reys, R. E., & Post, T. R. The Mathematics Laboratory, Theory to Practice. Boston: Prindle, Weber, & Schmidt, Inc., 1973.

Rouse, W. The Mathematics Laboratory: Misnamed, Misjudged, Misunderstood. School Science and Mathematics, January 1972, LXXII, 48-56.

Scanlon, R. Individually Prescribed Instruction: A System of Individualized Instruction. Educational Technology, December 1970, 10(12), 44-46.

Scriven, M. The Methodology of Evaluation. Perspectives of Curriculum Evaluation. R. Stake (Ed.), Chicago: Rand McNally & Company, 1967.

Shulman, L. S. Psychology and Mathematics Education. Mathematics Education. E. G. Begle (Ed.), 69th Yearbook of NSSE, Chicago: University of Chicago Press, 1970.

Sjogren, D. D. Achievement as a Function of Time. American Educational Research Journal, 4, 1967.

Skinner, B. F. The Science of Learning and the Art of Teaching. Harvard Educational Review, Spring 1954, XXIV, 86-97.

Sunde, A. Individualization in Mathematics Achievement. Unpublished Master's thesis, University of Alberta, 1970.

Suppes, P. The Use of Computers in Education. Scientific American, September 1966, 215, 206-221.

te Kampe, B. G. Individualized Instruction in Grade Seven Mathematics: The Teacher's Role. Unpublished Master's thesis, University of Alberta, 1970.

Walker, H. M., & Lev, J. Statistical Inference. New York: Henry Holt & Company, 1953.

Washburne, C. Adjusting the School to the Child. Yonkers-in-Hudson, New York: World Book Company, 1932.

Washburne, C. Winnekta. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1963.

Weber, P. N. A Study of the Relationships of Ability, Personality, and Motivation Factors with Achievement in Calculus via Two Instructional Treatments. Unpublished doctoral dissertation, University of Alberta, 1973.

Westrom, M. Individualization in Mathematics - A Description. Unpublished doctoral dissertation, University of Alberta, 1971.

Yeager, J. L., & Kissel, M. A. An Investigation of the Relationship between Selected Student Characteristics and Time Required to Achieve Unit Mastery. Working Paper No. 46, University of Pittsburgh, Learning Research and Development Center, 1969.

APPENDIX I

OUTLINE OF CHAPTER III OF SIR GEORGE SIMPSON JUNIOR HIGH SCHOOL GRADE NINE MATHEMATICS PROGRAM

UNIT III: ALGEBRAIC EXPRESSIONS (POLYNOMIALS)

A. Powers of Numbers (Exponential Notation)

1. Review
   - (a) Powers of numbers, e.g., powers of 10, of 5, etc.
   - (b) Base of powers
   - (c) Exponents (indices)
   - (d) Factors (prime, composite, common)

B. Integral Powers

1. Positive integral powers
   - (a) Introduce
     - (i) $x^m \cdot x^n$
     - (ii) $x^m \div x^n$
   - (b) Do
     - (i) $(x^m)^n$
     - (ii) $(xy)^m$
2. Zero powers
   - (a) Review
     - (i) Non-zero real numbers
     - (ii) 0^0
3. Negative integral powers
   - (a) Review
     - (i) $x^m \cdot x^n$
     - (ii) $x^m \div x^n$
     - (iii) $(x^m)^n$
     - (iv) $(xy)^m$
4. Computation with powers

5. Applications of base 10

   (a) Expanded form, e.g., 400.74

   (b) Scientific notation, e.g.,

   (i) $5321 = 5.321 \times 10^3$

   (ii) $5.32 = 5.32 \times 10^0$

   (iii) $.0053 = 5.3 \times 10^{-3}$

C. Expressions

1. Language of algebra

   (a) Variable

   (b) Term, constant term

   (c) Coefficients

   (d) Factors of products

2. Operations involved in expressions (e.g., taking the additive inverse of a number)

3. Order of operations (e.g., $3 + 4 \times 5$, $(3 + 4) \times 5$)

D. Polynomials

1. Meaning of polynomial

   (a) Monomial

   (b) Binomial

   (c) Trinomial

2. Degree of polynomial in one variable

   (a) Degree of polynomial

   (b) Linear polynomial

   (c) Quadratic polynomial

3. Similar terms

4. Replacement of variables with constants (evaluation of expressions)

E. Operations with Polynomials

1. Addition of polynomials

(a) Review

(i) Similar terms

(ii) Properties needed in finding the sum

(b) Do addition of

(i) Two monomials, e.g., $(6e) + (13e)$

(ii) Monomial and binomial, e.g., $(5) + (7x + 8)$

(iii) Two binomials, e.g., $(7x + 5) + (9x - 8)$

$(7x + 5) + (9x^2 - 8x)$

(iv) Other combinations of polynomials

2. Subtraction of polynomials

(a) Review--additive inverse

(b) Same as above for addition

3. Products of polynomials

(a) Two monomials

(b) Monomials and binomials

(c) Two binomials

(i) Any two binomials $(a + b)(c + d)$

(ii) Square of binomial $(a + b)^2$, $(a - b)^2$

(iii) Sum and difference $(a + b)(a - b) = a^2 - b^2$

4. Factoring of polynomials

(a) Removing a common factor which is a monomial, e.g., $2a^2 + 4ab$

(b) Removing a common factor which is a binomial,
e.g., $2ab + 8a + 6b + 24$

(c) Difference of squares (as in 3(c), (iii)).
(i) $x^2 - y^2$

(d) Trinomial squares
(i) $x^2 + 10x + 25 = (x + 5)^2$
(ii) $x^2 - 6x + 9 = (x - 3)^2$

(e) Trinomials
(i) Having 1 as the coefficient of the first term, e.g., $x^2 + 5x + 6$
(ii) Where a common factor can be removed first, e.g., $4x^2 + 20x + 24$
(iii) Having coefficient other than 1 for the first term, e.g., $2y^2 + 15y + 25$

5. Division of polynomials
(a) Review--reduction property
(b) Do division by
(i) The reduction property (factoring)
(ii) Long division

F. Rational Expressions

1. Rational expressions
(a) Meaning of rational expression
(b) Replacement of variables
(c) Meaningful replacements
(d) Reduction of quotients

2. Operations

(a) Multiplication

(i) Product property of quotients

(ii) Examples, problems

(b) Division

(i) Reciprocal property of quotients

(ii) Quotient property of quotients

(iii) Examples, problems

(c) Addition

(i) Review of L.C.M.

(ii) Sum property of quotients

(iii) Examples, problems

(d) Subtraction

(i) Review of difference property

(ii) Examples, problems

APPENDIX II

THE OBJECTIVE SHEETS

BASIC LEARNING TASK OBJECTIVES
SUB-UNIT I

OBJECTIVE I.1

To find the product of two monomial expressions.

Examples

A. Find the product of $2x$ and $3y$.

B. Evaluate $4a^2b \cdot 7a^3b^3$.

Solutions

A. $2x \cdot 3y = 2 \cdot 3 \cdot x \cdot y$
$= 6xy$

B. $4a^2b \cdot 7a^3b^3 = 4 \cdot 7 \cdot a^2a^3 \cdot b \cdot b^3$
$= 28a^5b^4$

OBJECTIVE I.2

To express a monomial expression as the product of two factors.

Examples

A. Find the missing factor $6x^2y^3 = (3xy) \cdot (\quad)$.

B. State two factors whose product is $10a^3b^2$.

Solutions

A. $6x^2y^3 = (3xy) \cdot (2xy^2)$

B. There are many possible solutions. For example:

(i) $2a^2b$, $5ab$
(ii) $10a^3$, b^2
(iii) $5ab^2$, $2a^2$

OBJECTIVE I.3

To find the product of a monomial and a binomial expression.

Examples

A. Find the product of $2x$ and $3y + 4x$.

B. Evaluate $4t^2(x - 5m)$.

Solutions

A. $2x \cdot (3y + 4x) = 2x \cdot 3y + 2x \cdot 4x$
$= 6xy + 8x^2$

B. $4t^2(x - 5m) = 4t^2 \cdot x - 4t^2 \cdot 5m$
$= 4t^2x - 20t^2m$

OBJECTIVE I.4

To express a binomial as the product of its greatest monomial factor and another binomial.

Examples

A. Express $3m^2 + 6mt$ as the product of its greatest monomial factor and another binomial.

B. Factor $4ab - 2b$.

Solutions

A. $3m^2 + 6mt = 3m \cdot m + 3m \cdot 2t$
$= 3m(m + 2t)$

B. $4ab - 2b = 2b \cdot 2a - 2b \cdot 1$
$= 2b(2a - 1)$

OBJECTIVE I.5

To find the product of a monomial and a trinomial expression.

Examples

A. Expand $2z(3t - 4zt + 1)$.

B. Evaluate $4w^2(wx - 2y - 3w^3)$.

Solutions

A. $2z(3t - 4zt + 1) = 2z \cdot 3t - 2z \cdot 4zt + 2z \cdot 1$
$= 6zt - 8z^2t + 2z$

B. $4w^2(wx - 2y - 3w^3) = 4w^2 \cdot wx - 4w^2 \cdot 2y - 4w^2 \cdot 3w^3$
$= 4w^3x - 8w^2y - 12w^5$

OBJECTIVE I.6

To express a trinomial as the product of its greatest monomial factor and another trinomial.

Examples

A. Factor $x^3 + 3x^2 - 4x$.

B. Find the greatest monomial factor of $6a^2y + 4ay^2 + 2a$ and write the trinomial as a product.

Solutions

A. $x^3 + 3x^2 - 4x = x \cdot x^2 + x \cdot 3x - x \cdot 4$
$= x(x^2 + 3x - 4)$

B. $6a^2y + 4ay^2 + 2a = 2a \cdot 3ay + 2a \cdot 2y^2 + 2a \cdot 1$
$= 2a(3ay + 2y^2 + 1)$

OBJECTIVE I.7

To express a binomial of the form $a(b + c) + d(b + c)$ as the product of two binomial factors.

Examples

A. Express $3q(y + 5) + 6(y + 5)$ as a product of two factors.

B. Factor $4x(2x - 1) - 3(2x - 1)$.

Solutions

A. $3q(y + 5) + 6(y + 5) = (3q + 6)(y + 5)$

B. $4x(2x - 1) - 3(2x - 1) = (4x - 3)(2x - 1)$

OBJECTIVE I.8

To find the product of two binomials of the form $(a + b)(c + d)$ where b and d are positive integers.

Examples

A. Find the product of $(2x + 5)$ and $(x + 3y)$.

B. Expand $(a + 2c)(2m + 3t)$.

Solutions

A. $(2x + 5)(x + 3y) = 2x(x + 3y) + 5(x + 3y)$

$= 2x \cdot x + 2x \cdot 3y + 5 \cdot x + 5 \cdot 3y$

$= 2x^2 + 6xy + 5x + 15y$

B. $(a + 2c)(2m + 3t) = a(2m + 3t) + 2c(2m + 3t)$

$= 2am + 3at + 4cm + 6ct$

OBJECTIVE I.9

To find the product of two binomials of the form $(a + b)(c + d)$ where b and d are negative integers.

Examples

A. Expand $(x - 5)(m - 3)$.

B. Evaluate $(2a - 3b)(t - a)$.

Solutions

A. $(x - 5)(m - 3) = x(m - 3) - 5(m - 3)$

$= x \cdot m - x \cdot 3 - 5 \cdot m + 5 \cdot 3$

$= xm - 3x - 5m + 15$

B. $(2a - 3b)(t - a) = 2a(t - a) - 3b(t - a)$

$= 2at - 2a^2 - 3bt + 3ab$

OBJECTIVE I.10

To find the product of two binomials of the form $(a + b)(c + d)$ where either b or d is a negative integer and the other is a positive integer.

Examples

A. What is the product of $(2q + 3t)$ and $(a - b)$.

B. Evaluate $(w - 6)(3 + 5y)$.

Solutions

A. $(2q + 3t)(a - b) = 2q(a - b) + 3t(a - b)$
$= 2qa - 2qb + 3ta - 3tb$

B. $(w - 6)(3 + 5y) = w(3 + 5y) - 6(3 + 5y)$
$= 3w + 5yw - 18 - 30y$

BASIC LEARNING TASK OBJECTIVES
SUB-UNIT II

OBJECTIVE II.1

To find the product of two binomials of the form $(x+a)(x+b)$.

Examples

A. Find the product of $(x+5)$ and $(x-4)$.

B. Expand $(m-6)(m-3)$.

Solutions

A. $(x+5)(x-4) = x(x-4) + 5(x-4)$
$= x^2 - 4x + 5x - 20$
$= x^2 + x - 20$

B. $(m-6)(m-3) = m(m-3) - 6(m-3)$
$= m^2 - 3m - 6m + 18$
$= m^2 - 9m + 18$

OBJECTIVE II.2

To find the product of two binomials of the form $(ax+b)(cx+d)$.

Examples

A. Expand $(2x+3)(x+1)$.

B. Evaluate $(3t-4)(2t+5)$.

Solutions

A. $(2x+3)(x+1) = 2x(x+1) + 3(x+1)$
$= 2x^2 + 2x + 3x + 3$
$= 2x^2 + 5x + 3$

B. $(3t-4)(2t+5) = 3t(2t+5) - 4(2t+5)$
$= 6t^2 + 15t - 8t - 20$
$= 6t^2 + 7t - 20$

OBJECTIVE II.3

To factor trinomials of the form $ax^2 + bx + c$ where $a=1$ and b and c are positive integers.

Examples

A. Factor $a^2 + 5a + 6$.

B. Factor $y^2 + 10y + 21$.

Solutions

A. Two numbers whose sum is 5 and product is $1 \cdot 6 = 6$ are 3 and 2.

$a^2 + 5a + 6 = a^2 + 3a + 2a + 6$
$= (a^2 + 3a) + (2a + 6)$
$= a(a+3) + 2(a+3)$
$= (a+2)(a+3)$

B. Two numbers whose sum is 10 and product is $1 \cdot 21 = 21$ are 3 and 7.

$y^2 + 10y + 21 = y^2 + 3y + 7y + 21$
$= (y^2 + 3y) + (7y + 21)$
$= y(y+3) + 7(y+3)$
$= (y+7)(y+3)$

OBJECTIVE II.4

To factor trinomials of the form $ax^2 + bx + c$, where b and c are positive integers.

Examples

A. Factor $6x^2 + 7x + 2$.

B. Factor $2t^2 + 13t + 15$.

Solutions

A. Two numbers whose sum is 7 and product is $6 \cdot 2 = 12$ are 3 and 4.

$$\begin{aligned} 6x^2 + 7x + 2 &= 6x^2 + 3x + 4x + 2 \\ &= (6x^2 + 3x) + (4x + 2) \\ &= 3x(2x+1) + 2(2x+1) \\ &= (3x+2)(2x+1) \end{aligned}$$

B. Two numbers whose sum is 13 and product is $2 \cdot 15 = 30$ are 3 and 10.

$$\begin{aligned} 2t^2 + 13t + 15 &= 2t^2 + 3t + 10t + 15 \\ &= t(2t+3) + 5(2t+3) \\ &= (t+5)(2t+3) \end{aligned}$$

OBJECTIVE II.5

To factor trinomials of the form $ax^2 + bx + c$, where a=1 and b is a negative integer and c is a positive integer.

Examples

A. Factor $c^2 - 3c + 2$.

B. Factor $y^2 - 8y + 15$.

Solutions

A. Two numbers whose sum is -3 and product is $1 \cdot 2 = 2$ are -2 and -1.

$$\begin{aligned} c^2 - 3c + 2 &= c^2 - 2c - c + 2 \\ &= (c^2 - 2c) - (c-2) \\ &= c(c-2) - (c-2) \\ &= (c-1)(c-2) \end{aligned}$$

B. Two numbers whose sum is -8 and product is $1 \cdot 15 = 15$ are -5 and -3.

$$\begin{aligned} y^2 - 8y + 15 &= y^2 - 5y - 3y + 15 \\ &= (y^2 - 5y) - (3y - 15) \\ &= y(y-5) - 3(y-5) \\ &= (y-3)(y-5) \end{aligned}$$

OBJECTIVE II.6

To factor trinomials of the form $ax^2 + bx + c$, where b is a negative integer and c is a positive integer.

Examples

A. Factor $5r^2 - 11r + 2$.

B. Factor $6x^2 - 17x + 5$.

Solutions

A. Two numbers whose sum is -11 and product is $5 \cdot 2 = 10$ are -10 and -1.

$$\begin{aligned} 5r^2 - 11r + 2 &= 5r^2 - 10r - r + 2 \\ &= (5r^2 - 10r) - (r-2) \\ &= 5r(r-2) - (r-2) \\ &= (5r-1)(r-2) \end{aligned}$$

B. Two numbers whose sum is -17 and product is $6 \cdot 5 = 30$ are -15 and -2.

$$\begin{aligned} 6x^2 - 17x + 5 &= 6x^2 - 15x - 2x + 5 \\ &= (6x^2 - 15x) - (2x-5) \\ &= 3x(2x-5) - (2x-5) \\ &= (3x-1)(2x-5) \end{aligned}$$

OBJECTIVE II.7

To factor trinomials of the form $ax^2 + bx + c$, where a=1 and b is a positive integer and c is a negative integer.

Examples

A. Factor $q^2 + 2q - 15$.

B. Factor $w^2 + 10w - 24$.

Solutions

A. Two numbers whose sum is 2 and product is $1 \cdot (-15) = (-15)$ are 5 and -3.

$$\begin{aligned} q^2 + 2q - 15 &= q^2 + 5q - 3q - 15 \\ &= (q^2 + 5q) - (3q + 15) \\ &= q(q+5) - 3(q+5) \\ &= (q-3)(q+5) \end{aligned}$$

B. Two numbers whose sum is 10 and product is $1 \cdot (-24) = (-24)$ are -2 and 12.

$$\begin{aligned} w^2 + 10w - 24 &= w^2 - 2w + 12w - 24 \\ &= (w^2 - 2w) + (12w - 24) \\ &= w(w-2) + 12(w-2) \\ &= (w+12)(w-2) \end{aligned}$$

OBJECTIVE II.8

To factor trinomials of the form $ax^2 + bx + c$, where b is a positive integer and c is a negative integer.

Examples

A. Factor $4x^2 + 7x - 2$.

B. Factor $6m^2 + 5m - 6$.

Solutions

A. Two numbers whose sum is 7 and product is $4 \cdot (-2) = (-8)$ are -1 and 8.

$$\begin{aligned} 4x^2 + 7x - 2 &= 4x^2 - x + 8x - 2 \\ &= (4x^2 - x) + (8x - 2) \\ &= x(4x-1) + 2(4x-1) \\ &= (x+2)(4x-1) \end{aligned}$$

B. Two numbers whose sum is 5 and product is $6 \cdot (-6) = (-36)$ are -4 and 9.

$$\begin{aligned} 6m^2 + 5m - 6 &= 6m^2 - 4m + 9m - 6 \\ &= (6m^2 - 4m) + (9m - 6) \\ &= 2m(3m-2) + 3(3m-2) \\ &= (2m+3)(3m-2) \end{aligned}$$

OBJECTIVE II.9

To factor trinomials of the form $ax^2 + bx + c$, where a=1 and b and c are negative integers.

Examples

A. Factor $e^2 - 3e - 4$.

B. Factor $y^2 - 3y - 18$.

Solutions

A. Two numbers whose sum is -3 and product is $1 \cdot (-4) = (-4)$ are -4 and 1.

$$\begin{aligned} e^2 - 3e - 4 &= e^2 - 4e + e - 4 \\ &= (e^2 - 4e) + (e - 4) \\ &= e(e-4) + (e-4) \\ &= (e+1)(e-4) \end{aligned}$$

B. Two numbers whose sum is -3 and product is $1 \cdot (-18) = (-18)$ are 3 and -6.

$$\begin{aligned} y^2 - 3y - 18 &= y^2 + 3y - 6y - 18 \\ &= (y^2 + 3y) - (6y + 18) \\ &= y(y+3) - 6(y+3) \\ &= (y-6)(y+3) \end{aligned}$$

OBJECTIVE II.10

To factor trinomials of the form $ax^2 + bx + c$, where b and c are negative integers.

Examples

A. Factor $10t^2 - 11t - 6$.

B. Factor $4a^2 - 8a - 21$.

Solutions

A. Two numbers whose sum is -11 and product is $10 \cdot (-6) = (-60)$ are -15 and 4.

$$\begin{aligned} 10t^2 - 11t - 6 &= 10t^2 - 15t + 4t - 6 \\ &= (10t^2 - 15t) + (4t - 6) \\ &= 5t(2t-3) + 2(2t-3) \\ &= (5t+2)(2t-3) \end{aligned}$$

B. Two numbers whose sum is -8 and product is $4 \cdot (-21) = (-84)$ are 6 and -14.

$$\begin{aligned} 4a^2 - 8a - 21 &= 4a^2 + 6a - 14a - 21 \\ &= (4a^2 + 6a) - (14a + 21) \\ &= 2a(2a+3) - 7(2a+3) \\ &= (2a-7)(2a+3) \end{aligned}$$

BASIC LEARNING TASK OBJECTIVES
SUB-UNIT III

OBJECTIVE III.1

To find the square of a binomial.

Examples

A. Square $(x+5)$.

B. Evaluate $(2q-3)^2$.

Solutions

A. $(x+5)^2 = x^2 + 2 \cdot x \cdot 5 + 5^2$
$= x^2 + 10x + 25$

B. $(2q-3)^2 = (2q)^2 - 2 \cdot 2q \cdot 3 + (-3)^2$
$= 4q^2 - 12q + 9$

You might also use the methods of multiplying two binomials.

OBJECTIVE III.2

To factor a trinomial which is a perfect square.

Examples

A. Factor $y^2 - 6y + 9$.

B. Factor $16m^2 + 24m + 9$.

Solutions

A. $y^2 - 6y + 9 = (y-3)^2$

B. $16m^2 + 24m + 9 = (4m+3)^2$

OBJECTIVE III.3

To find the product of the sum and difference of two terms.

Examples

A. Expand $(a+6)(a-6)$.

B. Evaluate $(2x-3)(2x+3)$.

Solutions

A. $(a+6)(a-6) = a^2 - 6^2$
$= a^2 - 36$

B. $(2x-3)(2x+3) = (2x)^2 - (3)^2$
$= 4x^2 - 9$

OBJECTIVE III.4

To factor a binomial which is the difference of two squares.

Examples

A. Factor $z^2 - 4$.

B. Factor $9p^2 - 16$.

Solutions

A. $z^2 - 4 = (z)^2 - (2)^2$
$= (z-2)(z+2)$

B. $9p^2 - 16 = (3p)^2 - (4)^2$
$= (3p-4)(3p+4)$

OBJECTIVE III.5

To factor a polynomial of 4 terms by grouping where no rearrangement of terms or sign alteration is necessary.

Examples

A. Factor $ax + 3x + ay + 3y$. B. Factor $4mt - 2mx + 6yt - 3yx$.

Solutions

A. $ax + 3x + ay + 3y = (ax+3x) + (ay+3y)$
$= x(a+3) + y(a+3)$
$= (x+y)(a+3)$

B. $4mt - 2mx + 6yt - 3yx = (4mt-2mx) + (6yt-3yx)$
$= 2m(2t-x) + 3y(2t-x)$
$= (2m+3y)(2t-x)$

OBJECTIVE III.6

To factor a polynomial of 4 terms by grouping and requiring sign alteration but not rearrangement of terms.

Examples

A. Factor $2t - 3tm - 4 + 6m$. B. Factor $4xy + 8xz - y - 2z$.

Solutions

A. $2t - 3tm - 4 + 6m = (2t-3tm) - (4-6m)$
$= t(2-3m) - 2(2-3m)$
$= (t-2)(2-3m)$

B. $4xy + 8xz - y - 2z = (4xy+8xz) - (y+2z)$
$= 4x(y+2z) - (y+2z)$
$= (4x-1)(y+2z)$

OBJECTIVE III.7

To remove a common factor from a trinomial and then factor the remaining trinomial of the form $ax^2 + bx + c$, $a=1$.

Examples

A. Factor $2x^2 + 10x - 12$. B. Factor $q^3 - 4q^2 + 3q$.

Solutions

A. $2x^2 + 10x - 12 = 2(x^2+5x-6)$
$= 2(x^2+6x-x-6)$
$= 2[x(x+6) - (x+6)]$
$= 2(x-1)(x+6)$

B. $q^3 - 4q^2 + 3q = q(q^2-4q+3)$
$= q(q-3)(q-1)$

OBJECTIVE III.8

To remove a common factor from a trinomial and then factor a remaining trinomial of the form $ax^2 + bx + c$.

Examples

A. Factor $8t^2 + 20t + 12$.

B. Factor $6m^2x - 5mx - 6x$.

Solutions

A. $8t^2 + 20t + 12 = 4(2t^2+5t+3)$
$= 4(2t+3)(t+1)$

B. $6m^2x - 5mx - 6x = x(6m^2-5m-6)$
$= x(3m+2)(2m-3)$

OBJECTIVE III.9

To remove a common factor from a trinomial and then factor the remaining trinomial as a perfect square.

Examples

A. Factor $3a^2 + 36a+108$.

B. Factor $4q^3m - 12q^2m + 9qm$.

Solutions

A. $3a^2 + 36a + 108 = 3(a^2+12a+36)$
$= 3(a+6)^2$

B. $4q^3m - 12q^2m + 9qm = qm(4q^2-12q+9)$
$= qm(2q-3)^2$

OBJECTIVE III.10

To remove a common factor from a binomial and then factor the remaining binomial as the difference of two squares.

Examples

A. Factor $8a^3 - 18a$.

B. Factor x^5-9x^3.

Solutions

A. $8a^3 - 18a = 2a(4a^2-9)$
$= 2a(2a-3)(2a+3)$

B. $x^5 - 9x^3 = x^3(x^2-9)$
$= x^3(x-3)(x+3)$

APPENDIX III

THE WORKSHEET, FORMATIVE TESTS, EXPERIENCES, AND REVIEW SHEET USED IN SUBUNIT I

WORKSHEET I

THIS WORKSHEET IS TO GIVE YOU A BIT OF PRACTICE ON THE OBJECTIVES OF SUBUNIT I. IF YOU LOOK AT THE EXAMPLES ON YOUR OBJECTIVE SHEETS YOU SHOULD BE ABLE TO DO EACH QUESTION. IF YOUR TEACHER DID NOT FINISH GOING OVER ALL THE OBJECTIVES IN CLASS IT STILL WILL BE HELPFUL TO YOU TO DO ALL THE QUESTIONS.

REMEMBER: TAKE A LOOK AT THE EXAMPLES FIRST.

1. a. Find the product of $4p$ and $5q^5$.
   b. Evaluate $3x^2y \cdot 6x^3y^2$.

2. a. What is the missing factor $(3ab)(__) = 12a^2b^3$.
   b. Find two factors whose product is $18m^3n^4$.

3. a. Find the product of $5q$ and $(3m+4q)$.
   b. Evaluate $3b^2(c-3d)$.

4. a. Express $5y^3 + 10y^2$ as the product of its greatest monomial factor and another binomial.
   b. Factor $4abc - 6bx$.

5. a. Expand $3m(4t-2mn+1)$.
   b. Evaluate $2x^2(x^2+2xy+y^2)$.

6. a. Factor $2p^3 + 4p^4 - 2p$.
   b. Find the greatest monomial factor of $9b^2c - 3bc^2 + 6b^2c^2$ and write the trinomial as a product.

7. a. Express $3t(x-y) - 4(x-y)$ as the product of two factors.
   b. Factor $4m^2(x+3n) + 5(x+3n)$.

8. a. Find the product of $(3a+2b)(x+7)$.
   b. Expand $(2q+r)(w+3y)$.

9. a. Expand $(t-6)(2c-m)$.
   b. Evaluate $(3y-5z)(2a-y)$.

10. a. What is the product of $(4a+1)$ and $(3b-2a)$?
    b. Evaluate $(y-2t)(4+3x)$.

ANSWERS

1. $20pq^5$
   $18x^5y^3$
2. $4ab^2$

many answers

3. $15qm + 20q^2$
   $3b^2c - 9b^2d$
4. $5y^2(y+2z)$
   $2b(2ac-3x)$
5. $12mt - 6m^2n + 3m$
   $2x^4 + 4x^3y + 2x^2y^2$
6. $2p(p^2+2p^3-1)$
   $3bc(3b-c+2bc)$
7. $(3t-4)(x-y)$
   $(4m^2+5)(x+3n)$
8. $3ax+21a+2bx+14b$
   $2qw+6qy+rw+3ry$
9. $2tc-tm-12c+6m$
   $6ay-3y^2-10an-5ya$
10. $12ab-8ac+3b-2c$
    $4y+3xy-8t-6tx$

FORMATIVE TEST I

NAME _____ DATE _____

THIS TEST IS INTENDED TO SHOW YOU HOW MUCH YOU HAVE UNDERSTOOD THE OBJECTIVES SO FAR. IT WILL ASSIST US IN FINDING OUT WHERE YOU NEED MORE HELP. IT WILL NOT COUNT TOWARDS YOUR FINAL GRADE.

THE TEST HAS 2 QUESTIONS ON EACH OF THE OBJECTIVES OF SUBUNIT I AND YOU SHOULD WORK OUT THE ANSWERS AND PLACE THEM IN THE SPACES PROVIDED.

		EXPERIENCE
1. Evaluate $2x \cdot 4x^2$.	_____	I.1.A.
2. Find the product of $3a^2b^3$ and $5ab^2$.	_____	
3. Find two monomial factors whose product is $10m^5$.	_____	I.2.A.
4. Find the missing factor. $(3ay^2)(__) = 12a^2y^4$.	_____	
5. Expand $3z(5z+6y)$.	_____	I.3.A.
6. Evaluate $a^2(3a^2-2a)$.	_____	
7. Factor $6xy + 9xz$.	_____	I.4.A.
8. Find the factors of $3a^3b^2 - 2ab^2$.	_____	
9. Find the product of $4t$ and (t^3-2t+3).	_____	I.5.A.
10. Expand $2ys(x+3y-2s)$.	_____	
11. Factor $3ax + 3ay - 3az$.	_____	I.6.A.
12. Factor $2c^2m - 4c^2m^2 - 2cm$.	_____	
13. Express $a(p+q) + b(p+q)$ as the product of two factors.	_____	I.7.A.
14. Factor $3m(2x+y) - t(2x+y)$.	_____	
15. Expand $(a+2)(x+y)$.	_____	I.8.A.
16. Evaluate $(3c+2x)(m+2x)$.	_____	
17. Find the product of $(x-2q)$ and $(t-m)$.	_____	I.9.A.
18. Evaluate $(2y-5)(z-3r)$.	_____	
19. Expand $(w+4z)(5-3y)$.	_____	I.10.A.
20. Evaluate $(2q-3p)(3y+1)$.	_____	

NOW TURN IN YOUR PAPER FOR MARKING

NOW THAT YOUR TEST HAS BEEN MARKED, YOU MAY FIND THAT YOU NEED SOME PRACTICE ON SOME OF THE OBJECTIVES. IF SO, YOU SHOULD PICK UP THE EXPERIENCE SHEETS CIRCLED ABOVE. THESE WILL HELP YOU TO OVERCOME YOUR DIFFICULTIES AND YOU MAY DO SOME AT HOME IF YOU WISH. IF YOU DID NOT MAKE ANY ERRORS YOU MAY WORK IN THE ACTIVITY CORNER OR HELP ANOTHER STUDENT WHO ASKS YOU FOR ASSISTANCE.

FORMATIVE TEST I.A

NAME _____ DATE _____

THIS TEST IS INTENDED TO SHOW YOU HOW MUCH YOU HAVE UNDERSTOOD THE OBJECTIVES SO FAR. IT WILL ASSIST US IN FINDING OUT WHERE YOU NEED MORE HELP. IT WILL NOT COUNT TOWARDS YOUR FINAL GRADE.

THE TEST HAS 2 QUESTIONS ON EACH OF THE OBJECTIVES OF SUB-UNIT I AND YOU SHOULD WORK OUT THE ANSWERS AND PLACE THEM IN THE SPACES PROVIDED.

		EXPERIENCE
1. Evaluate $5t^3 \cdot 3t$.	_____	I.1.B.
2. Find the product of $2m^3n$ and $7m^2n^2$.	_____	
3. Find two monomial factors whose product is $8x^3$.	_____	I.2.B.
4. Find the missing factor. $(__) \cdot (2b^2) = 14b^3a$.	_____	
5. Expand $2a(4a+5b)$.	_____	I.3.B.
6. Evaluate $x^3(2x^2-5x)$.	_____	
7. Factor $15pt + 10pm$.	_____	I.4.B.
8. Find the factors of $2c^3y^2 - 5c^2y^3$.	_____	
9. Find the product of $3x$ and (x^3+3x^2-2x).	_____	I.5.B.
10. Expand $4pc(2c-3p+2t)$.	_____	
11. Factor $2ma - 2xa + 2qa$.	_____	I.6.B.
12. Factor $10yb^2 + 10y^2b - 5yb$.	_____	
13. Express $x(r+t) + y(r+t)$ as the product of two factors.	_____	I.7.B.
14. Factor $3x(2y-z) - 4(2y-z)$.	_____	
15. Expand $(a+2)(b+c)$.	_____	I.8.B.
16. Evaluate $(2e+5m)(y+3d)$.	_____	
17. Find the product of $(c-3t)(x-y)$.	_____	I.9.B.
18. Evaluate $(3x-4)(t-2m)$.	_____	
19. Expand $(a+2q)(4-3y)$.	_____	I.10.B.
20. Evaluate $(5y-2c)(3x+2)$.	_____	

NOW TURN IN YOUR PAPER FOR MARKING

NOW THAT YOUR TEST HAS BEEN MARKED, YOU MAY FIND THAT YOU NEED SOME PRACTICE ON SOME OF THE OBJECTIVES. IF SO, YOU SHOULD PICK UP THE EXPERIENCE SHEETS CIRCLED ABOVE. THESE WILL HELP YOU TO OVERCOME YOUR DIFFICULTIES AND YOU MAY DO SOME AT HOME IF YOU WISH. IF YOU DID NOT MAKE ANY ERRORS YOU MAY WORK IN THE ACTIVITY CORNER OR HELP ANOTHER STUDENT WHO ASKS FOR ASSISTANCE.

FORMATIVE TEST I.B.

NAME _____ DATE _____

THIS TEST IS INTENDED TO SHOW YOU HOW MUCH YOU HAVE UNDERSTOOD THE OBJECTIVES SO FAR. IT WILL ASSIST US IN FINDING OUT WHERE YOU NEED MORE HELP. IT WILL NOT COUNT TOWARDS YOUR FINAL GRADE.

THE TEST HAS 2 QUESTIONS ON EACH OF THE OBJECTIVES OF SUB-UNIT I AND YOU SHOULD WORK OUT THE ANSWERS AND PLACE THEM IN THE SPACES PROVIDED.

		EXPERIENCE
1. Evaluate $2w^2 \cdot 5w^3$.	_____	I.1.C.
2. Find the product of $6xy^3$ and $3x^3y^2$.	_____	
3. Find two monomial factors whose product is $12p^6$.	_____	I.2.C.
4. Find the missing factor. $(2t^2m)(__) = 6t^3m^3$.	_____	
5. Expand $5r(3r+7t)$.	_____	I.3.C.
6. Evaluate $y^2(4y-3y^3)$.	_____	
7. Factor $9aw + 6aa$.	_____	I.4.C.
8. Find the factors of $4b^2x^3 - 3bx^2$.	_____	
9. Find the product of $2c$ and $(2c^2-4c-1)$.	_____	I.5.C.
10. Expand $2bw(4w+3b-x)$.	_____	
11. Factor $4pq - 4pr + 4pt$.	_____	I.6.C.
12. Factor $2x^2y^2 + 4x^3y - 2xy$.	_____	
13. Express $m(n+t) + x(n+t)$ as the product of two factors.	_____	I.7.C.
14. Factor $2a(3c+4) - 3b(3c+4)$.	_____	
15. Expand $(r+4)(t+y)$.	_____	I.8.C.
16. Evaluate $(2x+3m)(z+3m)$.	_____	
17. Find the product of $(y-3t)$ and $(x-z)$.	_____	I.9.C.
18. Evaluate $(2c-3)(x-4y)$.	_____	
19. Expand $(x+5y)(3-2t)$.	_____	I.10.C.
20. Evaluate $(3c-2d)(3x+1)$.	_____	

NOW TURN IN YOUR PAPER FOR MARKING

NOW THAT YOUR TEST HAS BEEN MARKED, YOU MAY FIND THAT YOU NEED SOME PRACTICE ON SOME OF THE OBJECTIVES. IF SO, YOU SHOULD PICK UP THE EXPERIENCE SHEETS CIRCLED ABOVE. THESE WILL HELP YOU TO OVERCOME YOUR DIFFICULTIES AND YOU MAY DO SOME AT HOME IF YOU WISH. IF YOU DID NOT MAKE ANY ERRORS YOU MAY WORK IN THE ACTIVITY CORNER OR HELP ANOTHER STUDENT WHO ASKS YOU FOR ASSISTANCE.

FORMATIVE TEST I.C

NAME _____ DATE _____

THIS TEST IS INTENDED TO SHOW YOU HOW MUCH YOU HAVE UNDERSTOOD THE OBJECTIVES SO FAR. IT WILL ASSIST US IN FINDING OUT WHERE YOU NEED MORE HELP. IT WILL NOT COUNT TOWARDS YOUR FINAL GRADE.

THE TEST HAS 2 QUESTIONS ON EACH OF THE OBJECTIVES OF SUB-UNIT I AND YOU SHOULD WORK OUT THE ANSWERS AND PLACE THEM IN THE SPACES PROVIDED.

1. Evaluate $3y^4 . 4y$. _____
2. Find the product of $5c^3z^3$ and $2c^2z$. _____
3. Find two monomial factors whose product is $6t^5$. _____
4. Find the missing factor. $(__)(4aw) = 8a^4w^2$. _____
5. Expand $3d(6d+5x)$. _____
6. Evaluate $z^3(2z^2-3z)$. _____
7. Factor $4ab + 6ac$. _____
8. Find the factors of $4x^2y - 5x^2y^2$. _____
9. Find the product of $3w$ and (w^3-4w^2+3). _____
10. Expand $3ax(t-2a+3x)$. _____
11. Factor $2xy + 2xy - 2wy$. _____
12. Factor $3cp^3 - 6c^2p^2 + 3cp$. _____
13. Express $z(d+w) + q(d+w)$ as the product of two factors. _____
14. Factor $4y(2b-c) - 3x(2b-c)$. _____
15. Expand $(q+3)(t+z)$. _____
16. Evaluate $(3a+4c)(x+2c)$. _____
17. Find the product of $(p-2q)$ and $(r-w)$. _____
18. Evaluate $(3y-2)(x-4z)$. _____
19. Expand $(m+3d)(4-3x)$. _____
20. Evaluate $(2b-3q)(2t+1)$. _____

NOW TURN IN YOUR PAPER FOR MARKING

EXPERIENCE I.1.A.

NAME _____ TIME STARTED ____ TIME FINISHED _____

FIND AN EXPRESSION IN THE SECOND COLUMN WHICH IS THE PRODUCT OF THE MONOMIALS IN EACH OF THE FOLLOWING. PUT YOUR ANSWERS IN THE 3 BY 3 SQUARE BELOW. QUESTIONS A AND F HAVE BEEN DONE FOR YOU.

COLUMN 1	COLUMN 2
A. 3, 2a	1. 6
B. 4a, 3a	2. $10ab^2$
C. 5ab, 2a	3. $12a^5b^3$
D. $2a^2b$, 5a	4. 6a
E. $10a^2b^2$, ab	5. $12a^5b^6$
F. $6a^2b^3$, $2a^3b^3$	6. $10a^2b$
G. $3a^3b^2$, $4a^3b$	7. $10a^3b^3$
H. $12a^4b$, ab^2	8. $12a^6b^3$
I. $2a^2b$, $6a^4b$	9. $10a^3b$
	10. $12a^6b^2$
	11. $12a^2$
	12. $10a^2b^6$
	13. 12a

A 4	B	C
D	E	F 5
G	H	I

NOW ADD THE FIGURES YOU HAVE ENTERED IN THE SQUARE, (1) HORIZONTALLY (ACROSS), (2) VERTICALLY (DOWN) AND (3) DIAGONALLY. IF YOU HAVE ANSWERED THE QUESTIONS CORRECTLY ALL THE TOTALS SHOULD BE 21.

EXPERIENCE I.1.B.

NAME _____ TIME STARTED TIME FINISHED

MULTIPLICATION OF MONOMIALS IS COMMUTATIVE, THAT IS $(a)(b) = (b)(a)$

Example 1 $(2t)(3t) = 2 \cdot t \cdot 3 \cdot t$

$= (2 \cdot 3)(t \cdot t)$

$= 6t^2$

Example 2 $(2x^2)(4x^3) = (2 \cdot x^2)(4 \cdot x^3)$

$= (2 \cdot 4)(x^2 \cdot x^3)$

$= 8x^{2+3}$

$= 8x^5$

Example 3 $(4xy^2)(3xy) = (4 \cdot 3)(x \cdot x)(y^2 \cdot y)$

$= (12)(x^2)(y^3)$

$= 12x^2y^3$

NOW TRY TO FIND THE PRODUCT OF EACH OF THE FOLLOWING PAIRS OF MONOMIALS.

1. $(3a)(4b)$ _____
2. $(2b)(6b)$ _____
3. $(5x^2)(3x^3)$ _____
4. $(4xy)(5x^2y^2)$ _____
5. $(3b^2c^2)(4b^3c)$ _____

NOW CHECK YOUR ANSWERS. IF YOU DID NOT GET THEM ALL CORRECT ASK SOMEONE FOR HELP.

ANSWERS: 1. $12ab$ 2. $12b^2$ 3. $15x^5$ 4. $20x^3y^3$ 5. $12b^5c^3$

EXPERIENCE I.1.C.

NAME _____ TIME STARTED _____ TIME FINISHED _____

HAVE YOU REVIEWED THE EXAMPLES ON OBJECTIVE I.3? DO YOU UNDERSTAND WHERE YOU WENT WRONG WITH EXPERIENCES I.1.A. AND I.1.B? IF NOT,-- THEN ASK SOMEONE TO HELP YOU! WHEN YOU FEEL YOU CAN FIND THE PRODUCT OF TWO MONOMIALS TRY THE FOLLOWING:

Find the product of:

1. $5(2x)$ _____
2. $8(3b)$ _____
3. $(7c)(4c)$ _____
4. $(3x)(5x)$ _____
5. $(2y)(6y)$ _____

NOW CHECK YOUR ANSWERS ON THE RIGHT HAND SIDE OF THE PAGE.

Now try these:

6. $(2x)(3y)$ _____
7. $(4a^2)(3a)$ _____
8. $(3ab)(2ab)$ _____
9. $(5a^2b)(3ab^3)$ _____
10. $(3x^3y^2)(4x^2y)$ _____

ANSWERS

1. $10x$
2. $24b$
3. $28c^2$
4. $15x^2$
5. $12y^2$
6. $6xy$
7. $12a^3$
8. $6a^2b^2$
9. $15a^3b^4$
10. $12x^5y^3$

IF YOU MADE MORE THAN ONE ERROR, PLEASE ASK SOMEONE TO HELP YOU. IF YOU GOT ALL TEN CORRECT, "GOOD SHOW, MAN!"

EXPERIENCE I.2.A.

NAME _____ TIME STARTED TIME FINISHED

LET'S PLAY A GAME. I'LL GIVE YOU A MONOMIAL AND ONE OF ITS FACTORS. YOU TRY TO FIND A SECOND FACTOR SO THAT THE TWO FACTORS MULTIPLIED TOGETHER GIVE MY MONOMIAL. YOU GET ONE POINT FOR EACH CORRECT ANSWER AND I GET ONE FOR EACH WRONG ANSWER. WHOEVER GETS 5 POINTS FIRST WINS. HERE WE GO!

MY MONOMIAL	MY FACTOR	YOUR FACTOR
$8x^3$	$2x$	_____
$12a^2b$	$3ab$	_____
$7pqr$	pq	_____
$24x^3y^2z$	$8xyz$	_____
$15y^5$	$3y^2$	_____

HOPE YOU DIDN'T SKUNK ME!

$3ax^2$	$3a$	_____
$18p^3x^5$	$6px^2$	_____
$22k^2yt$	$2kt$	_____
$14a^2b$	$14a^2b$	_____

WHAT WAS THE FINAL SCORE. HIP, HIP, HURRAH FOR THE WINNER!

$2^4 \cdot 4 \cdot 1^4 \cdot 3^2 \cdot 1^7 \cdot 5^3 \cdot 2^4 \cdot 3^2 \cdot 3^3 \cdot 11^7$ -

EXPERIENCE I.2.B.

NAME _____ TIME STARTED TIME FINISHED

ANY MONOMIAL CAN BE FACTORED INTO TWO FACTORS IN SEVERAL WAYS. FOR EXAMPLE: $6x^2y = \underline{3x \cdot 2xy}$ or $\underline{3x^2 \cdot 2y}$ or $\underline{6xy \cdot x}$ etc.

IN COLUMN I YOU WILL FIND 1 FACTOR OF $12a^2b^3$. DETERMINE THE SECOND FACTOR AND LOCATE IT IN COLUMN II.

	COLUMN I	COLUMN II
_____	1. $3ab$	A. $3a$
_____	2. $2a^2b^2$	B. $6ab^2$
_____	3. $4a^2b$	C. $3b^2$
_____	4. $3a$	D. $6ab$
_____	5. $2ab^2$	E. $4ab^2$
_____	6. $4ab^3$	F. $3a^2b^2$
_____	7. $3b^2$	G. $4ab^3$
		H. $4ab$
		I. $6b$

STILL HAVING PROBLEMS? FIND A FRIEND. EACH OF YOU WRITE DOWN ALL THE PAIRS OF FACTORS YOU CAN WHOSE PRODUCT IS $6m^2n^2$. CHECK EACH OTHER. WHO HAS THE MOST? THERE ARE 18 POSSIBLE PAIRS.

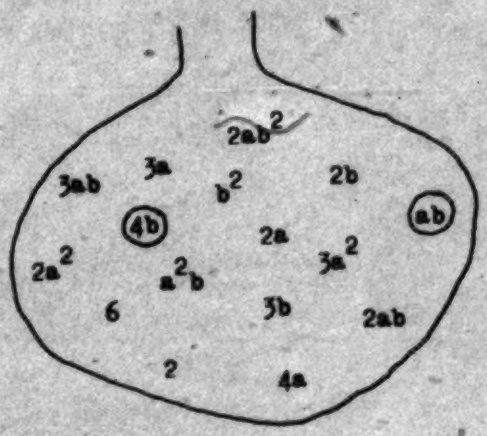
SCORES: 0 - 4 NEED HELP
5 - 9 PRETTY GOOD, KEEP PLUGING
10 - 14 YOU'RE IMPROVING--SOON WILL BE A PRO
15 - 18 A REAL EXPERT--TAKE A BOW

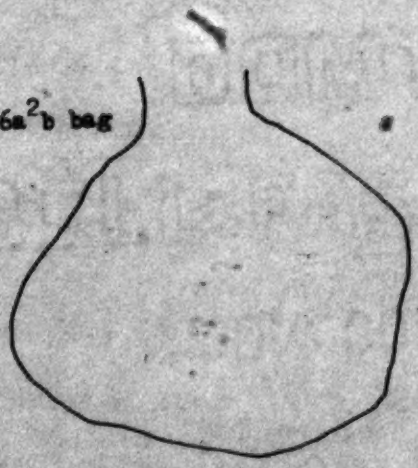
ANSWERS 1. E 2. I 3. C 4. G 5. D 6. A 7. H

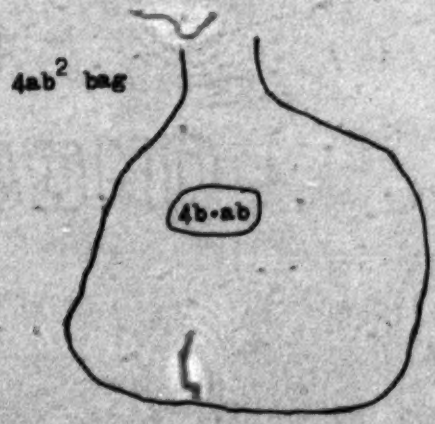
EXPERIENCE I.2.C.

NAME _____ TIME STARTED TIME FINISHED

JOHN HAD A BAG OF FACTORS AND FOR EVERY FACTOR IN THE BAG THERE WAS ANOTHER FACTOR WHICH TOGETHER WITH THE FIRST FACTOR WOULD GIVE EITHER THE PRODUCT $6a^2b$ OR THE PRODUCT $4ab^2$. SEE IF YOU CAN SORT THE FACTORS INTO PAIRS PLACING THOSE PAIRS WHOSE PRODUCT IS $6a^2b$ IN ONE BAG AND THOSE WHOSE PRODUCT IS $4ab^2$ IN A SECOND BAG.







ANSWERS
3ab·2a
$6 \cdot a^2b$
3a·2ab
$3b \cdot 2a^2$

ANSWERS
4b·ab
$b^2 \cdot 4a$
$2 \cdot 2ab^2$
2b·2ab

EXPERIENCE I.3.A.

NAME _____ TIME STARTED TIME FINISHED

Which of the following statements are true and which are false? Answer T or F as the case may be.

1. The product of b and $3a+2b$ is $3ab + 2b^2$. ____
2. The product of $2x$ and $2x+3xy$ is $4x + 6xy$. ____
3. The product of $3y$ and $4xy-5y$ is $12xy^2 + 15y^2$. ____
4. The product of $4a$ and $5a^2+4b^2$ is $20a^3 + 16ab^2$. ____
5. The product of $2p$ and $3pq-4p^2q^2$ is $6p^2q^2 - 8p^3q^3$. ____
6. The product of $3x^2y^3$ and $5xy^2 - 4x^3y^2$ is $15x^3y^5 - 12x^5y^5$ ____

Answers: 1. T 2. F 3. F 4. T 5. F 6. T

Did you get them all correct? If you had more than one wrong--ASK SOMEONE FOR HELP! If you had only ONE wrong, do you now see where you went wrong? If not, ASK FOR HELP!

EXPERIENCE I.3.B.

NAME _____ TIME STARTED TIME FINISHED

It looks as though you are having some trouble with this objective. Do you understand how to find the product of two monomials? (OBJECTIVE I.1)

In finding the product of a monomial and a binomial expression we use the distributive property.

For example, $2t(3t + 4) = 2t(3t) + 2t(4)$

$= 6t^2 + 8t$

Now try to complete the following:

$3x(5x + 2) = 3x(\quad) + 3x(2)$

$= $ _____

Did you get $15x^2 + 6x$? If not, ASK FOR HELP.

Now do the following exercises:

Find the product of each of the following monomials and binomials:

1. $4(x+2)$
2. $3y(2y+5)$
3. $5p(3-2p)$
4. $2y(3y^2+y)$
5. $5y^2(2y-3y^2)$
6. Complete: $7a^2(3a^3b+4a^2b^3) = 7a^2(3a^3b) + 7a^2(4a^2b^3)$

$= $ _____

7. Now for the "Gold Medal" question.

Evaluate: $3p^2q^3(5p^3q^2 - 7p^4q^3)$

Now check your answers. Turn the page upside down--you don't have to stand on your head!

Answers: 1. $4x+8$ 3. $15p-10p^2$ 5. $10y^3-15y^4$

2. $6y^2+15y$ 4. $6y^3+2y^2$ 6. $21a^5b+28a^4b^3$

7. $15p^5q^5-21p^6q^6$ Did you win the "Gold Medal"?

If so, stand to attention for "O, Canada"!

EXPERIENCE I.3.C.

NAME _____ TIME STARTED TIME FINISHED

Have you reviewed the examples on objective I.3? Do you understand where you went wrong with Experiences I.3.A. and I.3.B? If you do not feel you understand multiplication of a monomial and a polynomial ASK SOMEONE FOR HELP! Now, find these products:

1. $x(2x+3)$ _____
2. $b(3b+2c)$ _____
3. $2p(4p^2+3p^3)$ _____
4. $xy(3x^2y+2xy^2)$ _____
5. $2x(3x-4)$ _____
6. $3pq(5p^2+4q^2)$ _____
7. $3t^2(2t^2-4t)$ _____

Answers:
1. $2x^2+3x$
2. $3b^2+2bc$
3. $8p^3+6p^4$
4. $3x^3y^2+2x^2y^3$
5. $6x^2-8x$
6. $15p^3q+12pq^3$
7. $6t^4-12t^3$

EXPERIENCE I.4.A.

NAME _____ TIME STARTED TIME FINISHED

As a curler would say, you are now ready to play the second end. In the hack, ready, go!

We would like you to find factors in the second column which are equivalent to each of the expressions listed in the first column.

Column 1.	Column 2.
1. $a^2 + 2ab$	V. $5a^2(5a^2-3)$
2. $a^3 - a^2$	S. $2a^2(2+3a^2)$
3. $4a^2 + 6a^4$	T. $5a^2(5a^2+3)$
4. $25a^4 - 15a^2$	G. $a^2(a-1)$
5. $6a^3b + 3a^2b$	K. $3a^3b^2(2-b)$
6. $3a^3b^2 + 9a^2b^3$	I. $4ab^3(2ab-3)$
7. $8a^2b^4 - 12ab^3$	E. $a(a+2b)$
	R. $3a^2b^2(a+3b)$
	A. $3a^2b(2a+1)$

Answers: 1. E 2. G 3. S 4. V 5. A 6. R 7. I

So, the answers are E, G, S, V, A, R, and I. Now "scramble" these to find a famous St. Albertan, by Hec(k)!

EXPERIENCE I.4.3.

NAME _____ TIME STARTED TIME FINISHED

Multiplication of a binomial by a monomial produces a polynomial in which all the terms have a common factor that is equal to or is a multiple of the monomial. The reverse operation is called factoring and is simply the backwards application of the distributive law.

Example 1: Express $3x^3-6x$ as the product of a polynomial and a monomial.

Solution: The terms of $3x^3-6x$ have, as a common factor, $3x$ hence,

$$3x^3-6x = 3x(x^2) - 3x(2)$$
$$= 3x(x^2-2)$$

Express the following polynomials as products of binomials and monomials.

1. $2b + 4$ _____
2. $3t + 6t^2$ _____
3. $6x^2 - 15x^4$ _____
4. $7t^5 - 21t^3$ _____
5. $5a^2b^2 + 15a^3b^3$ _____
6. $8xy^3 - 4x^3y$ _____

Answers: 1. $2(b+2)$ 3. $3x^2(2-5x^2)$ 5. $5a^2b^2(1+3ab)$
2. $3t(1+2t)$ 4. $7t^3(t^2-3)$ 6. $4xy(2y^2-x^2)$

EXPERIENCE I.4.C.

NAME _____ TIME STARTED TIME FINISHED

You must be finding this factoring business rather tough! Have you looked over the examples on Objective I.4. and the Experiences I.4.A. and I.4.B.? Do you understand where you went wrong? If not, ASK SOMEONE FOR HELP!

Remember, when we are factoring expressions we want to "take out" the GREATEST COMMON FACTOR.

Let's try to factor a few polynomial expressions. Good Luck!

1. $2a + 6$
2. $2x + 3x^2$
3. $5b^3 + 3b^2$
4. $4t^3 + 8t^4$
5. $4a^4 + 6a^2$
6. $3x^2 - 5x^3$
7. $6p^2 - 4p^3$
8. $3ab + 2a^2b^3$
9. $5x^3y^2 - 10x^2y^3$

Answers:

1. $2(a+3)$
2. $x(2+3x)$
3. $b^2(5b+3)$
4. $4t^3(1+2t)$
5. $2a^2(2a^2+3)$
6. $x^2(3-5x)$
7. $2p^2(3-2p)$
8. $ab(3+2ab^2)$
9. $5x^2y^2(x-2y)$

EXPERIENCE I.5.A.

NAME _____ TIME STARTED TIME FINISHED

FIND AN EXPRESSION IN THE SECOND COLUMN WHICH IS EQUIVALENT TO THE INDICATED PRODUCT IN THE FIRST COLUMN.

COLUMN I	COLUMN II
____ 1. $4x(x+y+2z)$	I. $az^2 - 2a^2z^2 + 3a^3z^2$
____ 2. $3a^2(2x-a+1)$	S. $6a^2x + 6ax + ayz$
____ 3. $2y(4a+3x-2y)$	E. $4x^2 + 4xy + 8xz$
____ 4. $z^2(a-2a^2+3a^3)$	E. $2a^3xz^2 + 8ax^3z^2 + 6axyz^2$
____ 5. $yz(2y-3z+a)$	O. $12x^3y - 6ax^4y - 9ax^3yz$
____ 6. $3x^3y(4-2ax-3az)$	K. $8ay + 6xy - 4y^2$
____ 7. $2axz^2(a^2+4x^2+3y)$	L. $4ax^2 - 3ay^3 + 6ax^2y$
____ 8. $-a(6ax+6x+yz)$	S. $6a^2x - 3a^3 + 3a^2$
	M. $2y^2z - 3yz^2 + ayz$

DID YOU FIND A MYSTERY WORD? ARE THEY YOUR FAVORITE FOOTBALL TEAM?

EXPERIENCE I.5.B.

NAME _____ TIME STARTED TIME FINISHED

John: Let's look at an example first.

Sue: O.K. with me.

John: Multiply $2x^2$ and $3x - 4y + 2xy$.

Sue: Hey, that just means to find the product.

John: That's right! Here we go.

$$2x^2 \cdot (3x-4y+2xy) = (2x^2 \cdot 3x) - (2x^2 \cdot 4y) + (2x^2 \cdot 2xy)$$
$$= 6x^3 - 8x^2y + 4x^3y$$

Sue: Goodness gracious, that's just the distributive property in action! I can do the second line in my head.

John: Right again. Let's try some more examples just to make sure.

1. $3q \cdot (t+k-m) =$ _____
2. The product of $4y^2$ and $3z + 2a^2 + 2y$ is _____
3. $ab \cdot (4a^2-2ab+3b^2) =$ _____
4. $3tx(t^2-5-4tx^2) =$ _____
5. $3z^3(2+3z+4y^2z) =$ _____
6. $2xyz(x^2y-3y^2z+5zx^2) =$ _____

Sue: My answers are $3qt + 3qk - 3qm$, $12y^2z + 8a^2y^2 + 8y^3$, $4a^3b - 2a^2b^2 + 3ab^3$, $3t^3x - 15tx - 12t^2x^3$, $6z^3 + 9z^4 + 12y^2z^4$, and $2x^3y^2z - 6xy^3z^2 + 10x^3yz^2$.

John: Mine are the same. Chances are we have them all right.

EXPERIENCE I.5.C.

NAME _____ TIME STARTED TIME FINISHED

TO MULTIPLY A MONOMIAL BY A POLYNOMIAL OF 3 TERMS, YOU JUST MULTIPLY THE MONOMIAL BY EACH TERM OF THE POLYNOMIAL AND THEN ADD THE RESULTS. WATCH THIS!

$2ax(3a-4x+5)$

Multiply $2ax$ by $3a$ ⟶ $6a^2x$

Multiply $2ax$ by $-4x$ ⟶ $-8ax^2$

Multiply $2ax$ by 5 ⟶ $10ax$

Add the results $\underline{6a^2x - 8ax^2 + 10ax}$. This is your product.

REMEMBER THAT YOU CANNOT COMBINE UNLIKE TERMS.

$2ax(3a-4x+5) = 6a^2x - 8ax^2 + 10ax$

Now try these.

1. $5(p-2q+3k)$ = _____
2. $3y(2y^2+4y+3)$ = _____
3. $xt(5x-3x^2t+4mt)$ = _____
4. $x^3y^2(3x+2yz-5c)$ = _____
5. $2a^3bc(2bc-4ac^2-3a^2b^3)$ = _____

ANSWERS

1. $5p - 10q + 15k$
2. $6y^3 + 12y^2 + 9y$
3. $5x^2t - 3x^3t^2 + 4xmt^2$
4. $3x^4y^2 + 2x^3y^3z - 5x^3y^2c$
5. $4a^3b^2c^2 - 8a^4bc^3 - 6a^5b^4c$

EXPERIENCE I.6.A.

NAME _____ TIME STARTED TIME FINISHED

I'LL BET YOU REALLY MADE A SILLY MISTAKE ON THE TEST QUESTION SO IF YOU GET THIS ONE QUESTION CORRECT YOU CAN SKIP THE REST OF THIS EXPERIENCE. TRY THIS ONE. BE CAREFUL, IT MAY TRICK YOU.

FACTOR $12x^3y^2z^2 - 24xy^2z^3 + 4xy^2z$

THE ANSWER IS IN THE LOWER LEFT HAND CORNER. IF YOU MADE A MISTAKE YOU HAD BETTER TRY A FEW MORE.

Write each of the following as the product of the greatest common monomial factor and a trinomial.

1. $6x^3 - 12x^2 + 24x =$ _____
2. $xy + xz + xw =$ _____
3. $4ab - 12abc + 16a^2b^2c^2 =$ _____
4. $3t^2 + 6ty + 15t^2y^2 =$ _____
5. $10a^4 + 25a^3 - 5a^2 =$ _____
6. $8H^3K^2 - 4H^3K^3L^2 - 3HK^2L =$ _____

ANSWERS

$4ab(1-3c+4abc^2)$

$4xy^2z(3x^2z-6z^2+1)$

$3t(t+2y+5ty^2)$

$x(y+z+w)$

$HK^2(8H^2-4H^2KL^2-3L)$

$6z(z^2-2z+4)$

$5a^2(2a^2+5a-1)$

EXPERIENCE I.6.B.

NAME _____ TIME STARTED TIME FINISHED

WHEN YOU FOUND PRODUCTS OF EXPRESSIONS SUCH AS $4x(x^2+5x+6)$ YOU USED THE DISTRIBUTIVE PROPERTY AND DID THE FOLLOWING:

$$4x(x^2+5x+6) = 4x^3 + 20x^2 + 24x$$

WHAT IF YOU HAVE THE PRODUCT $4x^3 + 20x^2 + 24x$ AND WANT THE FACTORS? DID YOU SAY PUT THE DISTRIBUTIVE PROPERTY IN REVERSE AND GO THE OTHER WAY? IF YOU DID YOU ARE EXACTLY CORRECT.

$a(b+c+d) \longrightarrow ab + ac + ad$

and $ab + ac + ad \longrightarrow a(b+c+d)$

THE TRICK IS TO FIND "a" WHICH IS THE GREATEST MONOMIAL EXPRESSION THAT DIVIDES INTO EACH OF THE MONOMIAL TERMS OF THE TRINOMIAL EVENLY. IN THE ABOVE EXAMPLE "a" IS REPRESENTED BY 4x. SEE IF YOU CAN FILL IN THE FOLLOWING TABLE.

$ab + ac + ad$	a	$a(b+c+d)$
$3x^3 + 9x^2 + 12x$	$3x$	
$4a^3b - 12a^2bc + 8a^2c^2$		$4a^2(ab-3bc+2c^2)$
$15y^4 + 10xy^2 + 5x^2$		
$8p^2q - 12pq^2 + 16p^3t$	$4p$	$4p(2pq-3q^2+4p^2t)$
$6p - 12q + 30$		
$a^3b^3 + a^2b^4 - a^2b^3$		
$6y^3 + 12y^2 + 6y$		

ANSWERS

The missing greatest common factors are:
$4a^2$, 5, 6, a^2b^3, 6y.

The missing factored expressions are:
$3x(x^2+3x+4)$
$5(3y^4+2xy^2+x^2)$
$6(p-2q+5)$
$a^2b^3(a+b-1)$
$6y(y^2+2y+1)$

STILL HAVING TROUBLE?
MAYBE A FRIEND CAN
GIVE YOU SOME HELP!

EXPERIENCE I.6.C.

NAME _____ TIME STARTED TIME FINISHED

A FAMOUS FACTOR ONCE SAID "I AM THE GREATEST COMMON FACTOR". FOLLOWING EACH EXPRESSION, UNDERLINE THE COMMON FACTORS OF THE TERMS IN THE EXPRESSION AND CIRCLE THE GREATEST. THE FIRST ONE IS DONE FOR YOU.

1. $6a^2b + 9a^2b^2 + 12ab^2$	3a	6b	ab	(3ab)	a^2b
2. $12x^4 + 8x^3 + 6x^2$	4x	2	2x	$4x^2$	$2x^2$
3. $6ab + 9ac - 3ay$	3a	ab	ac	3	a
4. $p^3t^2 - 7p^2t^3 - 6pt^4$	pt	pt^2	p^2t	p	t
5. $3ab^2 + 6a^2b^2 - 9a^2b$	a^2b^2	$3a^2b^2$	3	3ab	3a
6. $5m^2 + 10m^4 + 15m^6$	m^2	$5m^2$	$5m^4$	m^6	m

NOW CHECK AND SEE IF YOUR GREATEST COMMON FACTORS ARE CORRECT. IF THEY ARE, NOW FACTOR EACH EXPRESSION ABOVE.

1. $6a^2b + 9a^2b^2 + 12ab^2 = 3ab(2a+3ab+4b)$
2.
3.
4.
5.
6.

1. $3ab(2a+3ab+4b)$
2. $2x^2(6x^2+4x+3)$
3. $3a(2b+3c-y)$
4. $pt^2(p^2-7pt-6t^2)$
5. $3ab(b+2ab-3a)$
6. $5m^2(1+2m^2+3m^4)$

1. — —⊖
2. — — ⊖
3. ⊖ — —
4. —⊖ — —
5. —⊖—
6. —⊖ —

EXPERIENCE I.7.A.

NAME _____ TIME STARTED TIME FINISHED

If you look at a binomial expression like $3a(2b+1) + 5c(2b+1)$ you will notice that each of the terms has the common factor $2b+1$. So, if we "take out" this common factor we would have $(2b+1)(3a+5c)$. Is this equivalent to $(3a+5c)(2b+1)$? Which property are we using here? (see answer below)

Now express the following as the product of two binomial factors.

1. $2x(3y+2) + 3y(3y+2) = (_____)(_____)$
2. $5a(2b+3c) + 2d(2b+3c) = (_____)(_____)$
3. $4p(2q-r) + 3s(2q-r) = (_____)(_____)$
4. $2m(5n+3) - 3p(5n+3) = (_____)(_____)$
5. $6x(3y-z) - 5t(3y-z) = (_____)(_____)$

Answers: We were using the commutative property.

1. $(3y+2)(2x+3y)$
2. $(2b+c)(5a+2d)$
3. $(2q-r)(4p+3s)$
4. $(5n+3)(2m-3p)$
5. $(3y-z)(6x-5t)$

EXPERIENCE I.7.B.

NAME _____ TIME STARTED TIME FINISHED

Hi there! It looks as though you're having some trouble expressing certain polynomials as the product of two binomial factors. I would like you to read over the examples for Objective I.7. and also Experience I.7.A. and then to complete each of the following sentences:

1. $3(2a+b) + 5c(2a+b) = (2a+b)(\quad)$
2. $2x(4y-z) + 3y(4y-z) = (\quad)(2x+3y)$
3. $5p(a+2c) - 3q(a+2c) = (5p-3q)(\quad)$
4. $3f(3c+d) - 4e(3c+d) = (\quad)(\quad)$
5. $4(2g-5h) - 3f(2g-5h) = (\quad)(\quad)$

Now check your answers. If you didn't get them all correct ASK SOMEONE FOR HELP! HELP! HELP!

Answers: 1. $(2a+b)(3+5c)$ 2. $(4y-z)(2x+3y)$

3. $(5p-3q)(a+2c)$ 4. $(3f-4e)(3c+d)$

5. $(4-3f)(2g-5h)$

Does it make any difference if we write the answer to number 5 as $(2g-5h)(4-3f)$?

EXPERIENCE I.7.C.

NAME _____ TIME STARTED TIME FINISHED

Hello to you again! I'm glad you haven't given up trying to understand how to write certain polynomials as the product of two binomial factors. Remember, "Rome wasn't built in a day!". Have you looked over the examples for Objective I.7. and Experiences I.7.A. and I.7.B.?

Answer true or false to the following. The answers are given on the right and you should cover them with a piece of paper, revealing each answer ONLY AS SOON AS YOU HAVE WRITTEN DOWN YOUR ANSWER. Any time you make a mistake try to find out how to arrive at the correct answer. Don't hesitate to ASK SOMEONE FOR HELP.

1. $2(a+b) + 3c(a+b) = (2+3c)(a+b)$ ____
2. $a(2c+d) + b(2c+d) = (a+2c)(b+d)$ ____
3. $3x(x+z) - y(x+z) = (3x+y)(x+z)$ ____
4. $2r(p-3q) + s(p-3q) = (2r+s)(p-3q)$ ____
5. $x(3f-2g) - y(3f-2g) = (x-y)(3f-2g)$ ____

ANSWERS:

1. T
2. F
3. F
4. T
5. T

Hope you had all five correct. Good Luck in the Formative Test I.C.

EXPERIENCE I.8.A.

NAME _____ TIME STARTED TIME FINISHED

Have you ever seen a 4 by 4 magic square? If you find an expression in the second column which is the expansion of each of the expressions in the first column, and insert the number in the appropriate square you should obtain a magic square.

Column 1	Column 2
B. $(a+c)(d+f)$	1. $6ad + 9af + 4cd + 6cf$
C. $(2a+c)(d+2f)$	2. $ad + af + cd + cf$
E. $(a+2c)(2d+3f)$	3. $2ad + 4af + cd + 2cf$
H. $(3a+c)(3d+2f)$	4. $4ad + 6af + 6cd + 9cf$
J. $(3a+3c)(2d+2f)$	5. $2ad + 3af + 4cd + 6cf$
K. $(3a+2c)(3d+2f)$	6. $9ad + 6af + 6cd + 4cf$
M. $(2a+3c)(2d+3f)$	7. $6ad + 6af + 6cd + 6cf$
P. $(3a+2c)(2d+3f)$	8. $9ad + 6af + 3cd + 2cf$

A 16	B	C	D 13
E	F 11	G 10	H
I 9	J	K	L 12
M	N 14	O 15	P

What totals do you get when you add,

(1) vertically

(2) horizontally

(3) diagonally?

Are they the same?

EXPERIENCE I.8.B.

NAME _____ TIME STARTED TIME FINISHED

Let's look at what happens when we multiply two binomials together. $(c+d)(e+f) = c(e+f) + d(e+f)$

$= ce + cf + de + df$

What we have done is to multiply every term in the first binomial by every term in the second binomial. This is using the distributive property. Try completing the following:

1. $(a+b)(c+d) =$ ac + ad + ____ + ____
2. $(2p+q)(r+s) =$ ____ + ____ + qr + qs
3. $(3m+2n)(4p+q) =$ 12mp ____ ____ ____
4. $(p+3q)(3r+2s) =$ ____ ____ ____ ____
5. $(r+4s)(4t+u) =$ ____ ____ ____ ____
6. $(2k+3m)(5n+2p) =$ ____ ____ ____ ____

Answers:

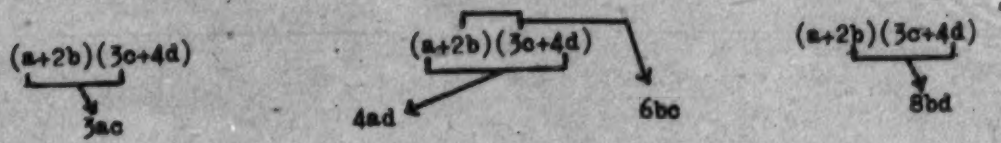
1. ac + ad + bc + bd
2. 2pr + 2ps + qr + qs
3. 12mp + 3mq + 8np + 2nq
4. 3pr + 2ps + 9qr + 6sq
5. 4rt + ru + 16st + 4us
6. 10kn + 4kp + 15mn + 6mp

EXPERIENCE I.8.C.

NAME _____ TIME STARTED TIME FINISHED

The FOIL Rule is a mechanical way of multiplying two binomials.

Consider the expression, $(a+2b)(3c+4d)$



product of First terms | product of Outside terms | product of Inside terms | product of Last terms

FOIL Rule:

The product of two binomials is the sum of the products of:

the First terms,

the Outside terms,

the Inside terms,

the Last terms.

You may want to use the FOIL Rule to find the product of the following:

1. $(a+b)(c+d)$ = _____
2. $(2x+3y)(x+z)$ = _____
3. $(3p+5q)(2r+s)$ = _____
4. $(c+3f)(d+2h)$ = _____
5. $(4z+w)(3z+2v)$ = _____

Answers: 1. $ac + ad + bc + bd$ 2. $2x^2 + 2xz + 3xy + 3yz$

3. $6pr + 3ps + 10qr + 5qs$ 4. $cd + 2ch + 3fd + 6fh$

5. $12z^2 + 8vz + 3wz + 2vw$

I'll bet you got them all right, this time!

EXPERIENCE I.9.A.

NAME _____ TIME STARTED TIME FINISHED

FOUR VARIABLES w, x, y, AND z LOVED THEIR MATHEMATICS CLASS BUT COULD NOT WAIT UNTIL FRIDAY NIGHT CAME AROUND. TO FIND OUT WHAT THEY HAD IN MIND, EXPAND THE FOLLOWING BINOMIALS AND PLACE THE LETTER IN FRONT OF THE CORRECT EXPANSION.

R.	$(2x-3z)(w-y)$	A	$wx - 2wy - 2xz + 4yz$
.	$(2z-3y)(w-x)$	____	$2wz - 2zx - 3yw + 3xy$
P.	$(2y-w)(x-3z)$	____	$2xy - 6yz - wx + 3wz$
A.	$(3w-2x)(z-2y)$	____	$3wz - 6wy - 2xz + 4xy$
Y.	$(x-w)(y-z)$	R	$2xw - 2xy - 3wz + 3yz$
T.	$(y-w)(z-3x)$	____	$yz - 3xy - wz + 3wx$
A.	$(w-2z)(x-2y)$	____	$xy - xz - wy + wz$

WHAT DO YOU THINK? WOULD YOU LIKE TO GO AND CELEBRATE YOUR NEW SUCCESS IN MATH!

EXPERIENCE I.9.B.

NAME _____ TIME STARTED TIME FINISHED

OBJECTIVE I.9 ASKS US TO MULTIPLY TWO DIFFERENCES TOGETHER. LET'S LOOK AT AN EXAMPLE.

multiply
difference difference
$(2x - y)$ $(3a - 2b)$

$= 2x(3a-2b) - y(3a-2b)$ ..... distributive property

$= (2x)(3a) + (2x)(-2b) - (y)(3a) - (y)(-2b)$ ..... distributive property

$= 6ax - 4bx - 3ay + 2by$

NOTICE THAT YOU CANNOT GO ANY FURTHER SINCE YOU HAVE UNLIKE TERMS. IF YOU CANNOT FOLLOW THIS EXAMPLE TAKE A GANDER AT THE OTHER EXAMPLES ON YOUR OBJECTIVE SHEET. IF YOU STILL ARE HAVING TROUBLE GET SOME HELP.

Now try to find these products.

1. $(m-t)(y-c) =$ _____
2. $(2x-3y)(x-z) =$ _____
3. $(4t-1)(2t-w) =$ _____
4. $(3a-5c)(4-a) =$ _____
5. $(2q-3y)(5t-4p) =$ _____
6. $(z-4x)(2x-3) =$ _____

THESE ANSWERS ARE SO LONG THAT WE WILL NOT PUT THEM UPSIDE DOWN ON THE PAPER.

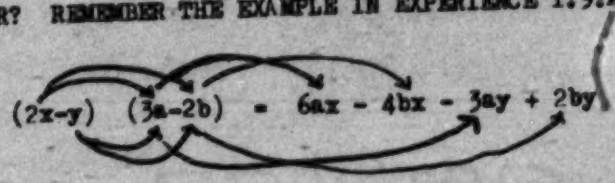
1. $my - mc - ty + tc$
2. $2x^2 - 2xz - 3xy + 3yz$
3. $8t^2 - 4tw - 2t + w$
4. $12a - 3a^2 - 20c + 5ac$
5. $10qt - 8qp - 15yt + 12py$
6. $2xz - 3z - 8x^2 + 12x$

REMEMBER THE ORDER OF THE TERMS DOES NOT MAKE A DIFFERENCE. DO YOU KNOW WHY?

EXPERIENCE I.9.C.

NAME _____ TIME STARTED TIME FINISHED

HAVE YOU NOTICED WHAT HAPPENS WHEN YOU MULTIPLY TWO BINOMIALS TOGETHER? REMEMBER THE EXAMPLE IN EXPERIENCE I.9.A.



LOOK the 6ax is just (2x) (3a)

the -4bx is just (2x) (-2b)

the -3ay is just (-y) (3a)

the 2by is just (-y) (-2b)

YOU MULTIPLY EVERY TERM IN THE FIRST BINOMIAL BY EVERY TERM IN THE SECOND BINOMIAL. THAT IS THE DISTRIBUTIVE PROPERTY.

I'LL HELP YOU A BIT WITH THESE FOR A WHILE.

1. $(q-x)(y-a) = ____ - qa - xy + ____$
2. $(2t-3r)(x-m) = 2tx - ____ - 3rx + ____$
3. $(4y-1)(2a-b) = ____ ____ ____ + b$
4. $(3q-5s)(4-t) = ____ - 3qt ____ ____$
5. $(m-4a)(2a-3) =$
6. $(2x-3y)(5z-4w) =$

ANSWERS

1. $qy - qa - xy + xa$
2. $2tx - 2tm - 3rx + 3rm$
3. $8ya - 4yb - 2a + b$
4. $12q - 3qt - 20s + 5ts$
5. $2am - 3m - 8a^2 + 12a$
6. $10xz - 8xw - 15yz + 12yw$

EXPERIENCE I.10.A.

NAME _____ TIME STARTED TIME FINISHED

HEY THERE--LAST EXPERIENCE BEFORE THE SECOND FORMATIVE TEST. BET YOU GET IT ALL RIGHT THIS TIME. HERE'S HOPING!!

SINCE THESE QUESTIONS ARE ALMOST THE SAME AS THOSE FOR THE PREVIOUS TWO OBJECTIVES, WE WILL JUST DO 4 MORE EXAMPLES. AWAY TO THE RACES.

Find the products or evaluate or expand. (these all mean the same)

1. $(m+4n)(q-r)$
2. $(3x-2y)(z-4w)$
3. $(a+4b)(2p-5q)$
4. $(6c-5d)(3a-7b)$

ANSWERS

$mq - mr + 4nq - 4nr$

$3xz + 12xw - 2yz - 8wy$

$2ap - 5aq + 8bp - 20bq$

$18ac - 42cb - 15ad + 35db$

EXPERIENCE I.10.B.

NAME _____ TIME STARTED TIME FINISHED

LET'S WORK UP TO THE SOLUTION. I'LL LEAVE A BIT MORE OF EACH QUESTION FOR YOU TO DO UNTIL YOU ARE ABLE TO DO IT ALL. CHECK YOUR ANSWERS AS YOU GO ALONG.

1. $(2x+y)(a-b) = 2x(a-b) + y(a-b) = 2xa - 2xb + ____ - ____$
2. $(3a-b)(m+2n) = 3a(m+2n) - b(m+2n) = 3am ____ - bm ____$
3. $(2t+3y)(p-4q) = 2t(p-4q) + _____ = ____ ____ + 3yp - 12yq$
4. $(a-2c)(y+5) = _____ - 2c(y+5) = ay + 5a ____ ____$
5. $(3q+5)(x-2y) = _____ _____ = 3qx ____ ____ - 10y$
6. $(2r-t)(3q+4a) = _____ _____ = ____ + 8ra - 3tq ____$
7. $(a+5b)(2c-3) = _____ _____ = ____ ____ ____ ____$
8. $(2m-3n)(2y-3x) = _____ _____ = ____ ____ ____ ____$

ANSWERS

1. $2x(a-b) + y(a-b) = 2xa - 2xb + (ya) - (yb)$
2. $3a(m+2n) - b(m+2n) = 3am + (6an) - bm - (2bn)$
3. $2t(p-4q) + (3y(p-4q)) - (2tp) - (8tq) + 3yp - 12yq$
4. $(a(y+5)) - 2c(y+5) = ay + 5a - (2cy) - (10c)$
5. $(3q(x-2y)) + (5(x-2y)) = 3qx - (6qy) + (5x) - 10y$
6. $(2r(3q+4a)) - (t(3q+4a)) = (6rq) + 8ra - 3tq - (4ta)$
7. $a(2c-3) + 5b(2c-3) = 2ac - 3a + 10bc - 15b$
8. $2m(2y-3x) - 3n(2y-3x) = 4my - 6mx - 6ny + 9nx$

EXPERIENCE I.10.C.

NAME _____ TIME STARTED TIME FINISHED

DID YOU COME ACROSS THE FOIL TRICK IN ANY OF THE EXPERIENCES YET? MAYBE IT CAN HELP YOU TO EXPAND THESE BINOMIALS SUCH AS $(a+b)(c-d)$.

F ⟶ MULTIPLY FIRST TWO ⟶ a AND c
O ⟶ MULTIPLY OUTSIDE TWO ⟶ a AND -d
I ⟶ MULTIPLY INSIDE TWO ⟶ b AND c
L ⟶ MULTIPLY LAST TWO ⟶ b AND -d

$$(a+b)(c-d) = \underset{F}{ac} - \underset{O}{ad} + \underset{I}{bc} - \underset{L}{bd}$$

Fill in the table to expand the following pairs of binomials.

Binomials	F	O	I	L	Product
$(p+q)(x-y)$		$-py$	qx		
$(2k-m)(a+4)$				$-4m$	
$(y+3z)(b-2a)$	yb		$3zb$		
$(2t-5)(3r+2)$					$6tr + 4t - 15r - 10$
$(4m+3x)(2y-3m)$					
$(r-w)(2y+a)$					
$(3c+2y)(4m-3a)$					

GET SOMEONE TO CHECK YOUR ANSWERS FOR YOU. SHOULD YOU REQUIRE SOME HELP THEY MAY BE ABLE TO ASSIST YOU.

REVIEW SHEET I

THE REVIEW SHEET IS GIVEN FOR TWO REASONS. FIRST, IF YOU MASTERED AN OBJECTIVE EARLY IN THE SUBUNIT IT MAY HAVE SLIPPED YOUR MIND. SECONDLY, PERHAPS YOU HAD QUITE A BIT OF TROUBLE AND NEED A BIT OF EXTRA PRACTICE. IN EITHER CASE THESE QUESTIONS WILL LET YOU KNOW HOW YOU ARE DOING.
GOOD LUCK!
NOTE: THE QUESTIONS ARE NOT IN THE SAME ORDER AS YOUR OBJECTIVES. CAN YOU MATCH THEM UP?

Find the following products.

1. $(3t^2)(5at)$
2. $4a^2m^3 \cdot 5am^2$
3. $3x(3y+4z)$
4. $3y^2(2y-5)$
5. $5q(q^2-2q+3)$
6. $2y^2(py+q+3y^2)$
7. $(x+5)(2y+z)$
8. $(2a+3b)(c+2x)$
9. $(q-3)(2r-y)$
10. $(3m-2p)(t-2x)$
11. $(x+2y)(t-4)$
12. $(5w-3)(2x+3)$

Find the missing factor.

13. $(3a)(___) = 15a^2b$
14. $(___)(4p^3q) = 12p^5q^3$

Factor.

15. $4r^2 + 8rt$
16. $12a^2b - 9ab^2$
17. $9m^3 + 6m^2 - 3m$
18. $4p^2q - 6pq^2 - 8p$
19. $a(5+x) - 3(5+x)$
20. $2y^2(a^2+c^2) - z^2(a^2+c^2)$

ANSWERS

1. $15at^3$
2. $20a^3m^5$
3. $9xy + 12xz$
4. $6y^3 - 15y^2$
5. $5q^3 - 10q^2 + 15q$
6. $2py^3 + 2qy^2 + 6y^4$
7. $2xy + xz + 10y + 5z$
8. $2ac + 4a^2 + 3bc + 6ab$
9. $2qr - qy - 6r + 3y$
10. $3mt - 6mx - 2pt + 4px$
11. $at - 4x + 2yt - 8y$
12. $10wx + 15w - 6x - 9$
13. $5ab$
14. $3p^2q^2$
15. $4r(r+2t)$
16. $3ab(4a-3b)$
17. $3m(3m^2+2m-1)$
18. $2p(2pq-3q^2-4)$
19. $(a-3)(5+x)$
20. $(2y^2-z^2)(a^2+c^2)$

APPENDIX IV

SUMMATIVE AND POSTSUMMATIVE TESTS

SUMMATIVE TEST

THIS TEST IS TO DETERMINE HOW WELL YOU HAVE MASTERED THE MATHEMATICS YOU HAVE STUDIED FOR THE PAST FOUR WEEKS. THERE IS ONE QUESTION ON EACH OF THE 30 OBJECTIVES STUDIED AND YOU ARE TO SUPPLY THE ANSWER ONLY, IN THE SPACE PROVIDED. YOU MAY DO YOUR WORK ON SCRATCH PAPER. TAKE YOUR TIME AND BE CAREFUL. GOOD LUCK!

1. Evaluate $2ab^2 \cdot 3ab^3$ _____
2. Find the missing factor $8m^2t^3 = (2mt)(\ ?\ )$ _____
3. Find the product $3x^2(y-2x)$ _____
4. Factor by removing a common factor $6rt - 3t$ _____
5. Expand $2a(ac-3ab+2)$ _____
6. Factor by removing a common factor $10y^3 - 8y^2z + 6y$ _____
7. Factor $2m(3q-1) + 3(3q-1)$ _____
8. Expand $(2a+3)(b+2c)$ _____
9. Find the product $(x-2y)(z-x)$ _____
10. Evaluate $(m-t)(p+r)$ _____
11. Expand $(x+3)(x-5)$ _____
12. Evaluate $(2y-3)(3y-4)$ _____

Factor the following trinomial expressions:

13. $t^2 + 6t + 8$ _____
14. $6m^2 + 7m + 2$ _____
15. $r^2 - 7r + 6$ _____
16. $3x^2 - 11x + 6$ _____
17. $a^2 + 3a - 18$ _____
18. $8q^2 + 10q - 3$ _____
19. $c^2 - c - 20$ _____
20. $5m^2 - 7m - 6$ _____

21. Square $(3r+2)$ _____

22. Factor as a perfect square $z^2 - 12z + 36$ _____

23. Expand $(t+5)(t-5)$ _____

24. Factor as the difference of squares $4y^2 - 49$ _____

Group and then factor the following polynomials:

25. $mt - xt + mr - xr$ _____

26. $6yz + 4az - 3y - 2a$ _____

Factor by removing a common factor and then factoring the remaining trinomial:

27. $m^3 - 7m^2 + 10m$ _____

28. $12y^2 + 21y - 6$ _____

Factor by removing a common factor and then factoring the remaining trinomial as a perfect square:

29. $9a^2b - 12ab + 4b$ _____

Factor by removing a common factor and then factoring the remaining binomial as the difference of squares:

30. $3at^2 - 12a$ _____

POSTSUMMATIVE TEST

THIS TEST IS TO DETERMINE HOW WELL YOU HAVE REMEMBERED THE MATHEMATICS WHICH YOU STUDIED SEVERAL WEEKS AGO. THERE IS ONE QUESTION ON EACH OF THE 30 OBJECTIVES STUDIED AND YOU ARE TO PROVIDE THE ANSWER ONLY, IN THE SPACE PROVIDED. YOU MAY DO YOUR WORK ON SCRATCH PAPER. TAKE YOUR TIME AND BE CAREFUL. GOOD LUCK!

1. Evaluate $3x^2y \cdot 4x^3y$ _____
2. Find the missing factor $6a^3t^2 = (3a^2t)(\ ?\ )$ _____
3. Find the product of $2m^2(p-3q)$ _____
4. Factor by removing a common factor $8ab - 4b$ _____
5. Expand $3y(yw-2yz+3)$ _____
6. Factor by removing a common factor $8t^3 - 6t^2r + 4t$ _____
7. Factor $3q(2w-1) + 2(2w-1)$ _____
8. Expand $(2m+5)(x+2c)$ _____
9. Find the product $(r-3y)(a-r)$ _____
10. Evaluate $(c-t)(q+z)$ _____
11. Expand $(m+2)(m-7)$ _____
12. Evaluate $(3d-2)(4d-3)$ _____

Factor the following trinomial expressions:

13. $y^2 + 5y + 6$ _____
14. $6n^2 + 11n + 3$ _____
15. $t^2 - 9t + 8$ _____
16. $2w^2 - 7w + 6$ _____
17. $b^2 + 5b - 24$ _____
18. $10r^2 - 13r - 3$ _____
19. $y^2 - y - 12$ _____
20. $3m^2 - 7m - 6$ _____

21. Square $(2x+3)$ _____

22. Factor as a perfect square $t^2 - 10t + 25$ _____

23. Expand $(z+4)(z-4)$ _____

24. Factor as the difference of squares $36x^2 - 49$ _____

Group and then factor the following polynomials:

25. $ab - cb + ax - cx$ _____

26. $10rt + 4mt - 5r - 2m$ _____

Factor by removing a common factor and then factoring the remaining trinomial:

27. $y^3 - 6y^2 + 8y$ _____

28. $8c^2 + 22c - 6$ _____

Factor by removing a common factor and then factoring the remaining trinomial as a perfect square:

29. $4m^2t - 12mt + 9t$ _____

Factor by removing a common factor and then factoring the remaining binomial as the difference of squares:

30. $2xa^2 - 18x$ _____

ITEM ANALYSIS FOR

	SUMMATIVE TEST		POSTSUMMATIVE TEST	
ITEM	DIFFICULTY	BISERIAL CORR	DIFFICULTY	BISERIAL CORR
1	.82	.71	.89	.62
2	.91	.67	.92	.82
3	.78	.63	.80	.51
4	.69	.75	.76	.85
5	.79	.74	.86	.69
6	.57	.65	.63	.58
7	.65	.70	.77	.88
8	.65	.58	.77	.66
9	.49	.80	.64	.87
10	.57	.81	.62	.77
11	.55	.71	.55	.78
12	.46	.73	.52	.73
13	.83	.92	.84	.77
14	.68	.91	.74	.85
15	.55	.93	.65	.99
16	.53	.93	.56	.95
17	.64	.85	.69	.94
18	.40	.88	.50	.89
19	.43	.90	.56	.92
20	.49	.95	.60	.85
21	.64	.81	.62	.84
22	.68	.75	.68	.84
23	.64	.73	.77	.78
24	.57	.83	.76	.99
25	.77	.82	.84	.84
26	.61	.82	.67	.82
27	.51	.91	.58	.91
28	.36	.90	.46	.82
29	.53	.90	.52	.81
30	.48	.80	.65	.90
	N = 152		N = 153	
	$\bar{X}$ = 18.2		$\bar{X}$ = 20.4	
	s^2 = 75.9		s^2 = 69.1	
	K-R 20 = .92		K-R 20 = .92	

APPENDIX V

RAW DATA COLLECTED

VARIABLES

1. Identification
2. SCAT percentile score
3. STEP percentile score
4. Previous achievement
5. Last formative test subunit I
6. Last formative test subunit II
7. Last formative test subunit III
8. Summative test
9. Postsummative test

1	2	3	4	5	6	7	8	9
00113	17	28	3	10	8	12	23	28
00213	97	76	4	16	20	16	28	28
00313	22	21	1	3	2	4	8	2
00413	95	46	5	20	20	20	30	30
00513	48	63	4	14	13	16	21	25
00613	29	38	1	15	12	12	15	21
00713	77	72	3	13	12	11	15	19
00813	24	26	3	8	2	4	12	11
00913	38	28	1	3	1	0	0	1
01013	36	58	4		2	2	5	16
01113	24	54	2	12	1	9		13
01213	73	58	2	12	0	3	7	9
01313	73	50	3	14	4	7	13	19
01413	15	13	2		0	0	6	4
01513	88	84	4	19	19	17	28	29
01613	29	46	3	15	7	10	18	9
01713	56	42	4	18	9	11	13	27
01813	40	50	2	1	3	3	1	3
01913	93	72	3	12	20	13	27	28
02013	80	58	4	16	11	11	21	17
02113	27	15	3	8	2	10	11	15
02213	60	50	3	12	16	18	22	25
02313	1	3	2	5	0	0	0	4
02413	11	19	2	19	6	12	23	22
02513	95	96	3	6	2	0	4	9
02613	73	13	5	18	19	19	29	30
02713	52	54	4	19	20	15	28	26
02821	77	72	3	20	19	13	24	27
02921	42	46	2	10	6	10	11	9
03021	48	54	3	18	16	18	21	29
03121	77	63	3	15	11	20	27	27
03221	19	15	2	2	0	2	2	4
03321	75	67	5	20	16	17	29	27
03421	79	76	4	16	8	18	26	30

1	2	3	4	5	6	7	8	9
03521	40	28	2	7	3	4	6	6
03621	77	84	5	20	20	19	27	27
03721	20	28	2	12	6	11	6	21
03821	56	31	3	12	4	11	15	16
03921	75	58	2	16	4	8	10	20
04021	82	88	3	12	0	10	16	22
04121	75	28	4	11	3	13	15	12
04221	97	42	5	13	18	9	20	25
04321	24	3	3	10	2	4	8	12
04421	19	23	4	19	17	20	30	28
04521	22	28	3	7	0	10	7	15
04621	75	88	4	20	18	17	22	24
04721	90	94	5	19	20	20	30	28
04821	70	46	4	14	5	17	21	24
04921	12	8	2	11	0	13	7	14
05021	48	67	3	20	0	12	20	25
05121	80	84	3	19	18	17	27	28
05221	83	67	5	18	1	10	21	29
05321	44	91	3	12	8	11	26	24
05432	25	31	3	19	11	15	20	24
05532	17	8	2		6	6	11	20
05632				20	11	16	24	29
05732	48	88	3	6	0	6	12	10
05832	83	91	4	16		15	19	21
05932	58	50	4	17	19	20	28	28
06032	72	58	4	8	7	15	29	28
06132	46	67	3	8	5	6	15	11
06232	79	46	3	9	0	5	11	14
06332	56	67	4	18	17	13	25	27
06432	36	50	2	3	0	1	10	19
06532	52	63	3	8	13		15	10
06632	94	58	3	6	8	12	25	24
06732	90	94	5	18	19	11	24	22
06832	54	46	4	9	3	7	15	14

1	2	3	4	5	6	7	8	9
06932	24	6	3	17	5	5	9	19
07032	75	19	4	11	18	14	22	26
07132	30	2	2	0	11	7	5	18
07232	30	13	3	16	3	1	10	10
07332	40	42	4	11	7	9	20	21
07432	60	54	3	18	11	11	22	25
07532	82	94	5	14	18	19	27	26
07632	15	19	3	6	0	2	7	7
07732	82	76	2	11	3	0	8	6
07832	36	17	3	10	13	14	22	30
07944	70	54	2	17	16	18	15	26
08044	80	76	5	18	16	20	27	29
08144	96	88	5	12	18	19	26	29
08244	46	58	3	20	20	18	26	28
08344	22	23	2	8	13	16	17	20
08444	98	84	5	19	12	20	30	30
08544	66	38	4	12	19	19	23	29
08644	40	67	3		9	13	21	23
08744	70	31	4	12	17	19	29	24
08844	32	54	3	16	14	8	28	29
08944	90	50		9	17	20	30	27
09044	56	63	3	7	0	9	12	11
09144				1	0	0	7	1
09244	16	10	2	11	9	12	24	26
09344	30	50	2	9	0	3	18	19
09444	13	17	3	11	13	4	10	16
09544	62	54	4	20	19	20	23	29
09644	29	35	2	7	6	4	13	14
09744	25	31	3	16	17	20	25	29
09844	83	88	4	20	17	19	26	25
09944	50	54	4	20	20	19	30	30
10044	36	38	5	16	19	20	30	30
10144	70	63	4	9	13	18	28	27
10244	62	50	3	11	18	12	24	25

1	2	3	4	5	6	7	8	9
10352	15	21	1	11	1		5	4
10452	75	67	4	20	20	14	23	29
10552	24	1	3	15	11	6	8	20
10652.	62	21	3	14	15	15	23	29
10752	44	21	4	16	16	12	23	24
10852	46	50	1	10	6	6	10	13
10952	42	26	3	9	2	4	14	19
11052	56	54	3	11	18	13	28	24
11152	8	42	1	11	19	0	5	10
11252	95	97	5	18	15	20	28	30
11352	68	84	3	17	18	16	29	29
11452	32	67	2	16	1	3	9	8
11552	9	15	2	14	14	9	13	16
11652	50	28	2	8	9	12	20	14
11752	30	28	3	13	9	9	13	12
11852	54	94	3	10	0	3	10	26
11952			3	18	18	12	17	28
12052	29	34	3	16	7	6	17	20
12152	99	84	5	14	20	18	22	26
12252	48	63	4	19	20	20	30	30
12352	88	97	3	12	7	7	5	12
12452	42	58	4	19	17	16	23	26
12552	58	23	3	16	17	11	18	20
12652	62	63	3	15	13	11	19	23
12752	70	84	3	14	11	9	21	20
12852	46	76	2	16	11	14	16	20
12961	79	67	4	17	20	19	27	28
13061	94	76	5	17	15	20	30	30
13161	86	88	4	20	20	19	29	27
13261	46	38	3	14	20	20	28	24
13361	79	80	3	14	20	18	30	26
13461	96	71	4	18	17	20	29	29
13561	25	19	1	10	6	10	9	6
13661	70	21	2	7	4	19	3	8

1	2	3	4	5	6	7	8	9
13761	60	35	3	16	7	18	7	10
13861	96	94	5	20	19	19	29	30
13961	29	26	2	7	2	6	11	10
14061	93	97	5	19	19	20	28	30
14161	68	46	3	20	18	20	27	30
14261	27	15	2	4	2	17	3	21
14361	13	1	2	9	6	8	12	17
14461	58	38	3	13	11	4	4	6
14561	72	58	3	19	15	20	27	22
14661	64	54	3	18	12	14	19	17
14761	50	31	1	8	3	8	10	11
14861				19	20	18	27	30
14961	19	46	1	6	2	0	0	0
15061	52	38	3	20	16	20	12	20
15161	52	80	4	11	3	19	28	28
15261	40	28	3	8	5	8	12	14
15361	60	46	3	18	20	19	24	27

APPENDIX VI

TABLES INDICATING DIFFERENCES IN ACHIEVEMENT ON EACH SUBUNIT WITHIN THE MMM AND NNN GROUPS AND THE ADJUSTED ACHIEVEMENT MEANS

DIFFERENCES IN ACHIEVEMENT ON EACH SUBUNIT BETWEEN THE LAST FORMATIVE TEST DURING THE SUBUNIT AND THE SUMMATIVE TEST WITHIN THE MMM AND NNN GROUPS

	N	$\bar{X}$(SU) (max=20)	SD(SU)	$\bar{X}$(ST) (max=10)	SD(ST)	r	t
MMM SU I	51	14.2	5.00	6.75	2.86	.73	1.28
SU II	51	10.3	7.45	5.73	3.72	.76	-1.63
SU III	51	14.1	5.68	5.94	3.49	.65	2.86**
NNN SU I	50	13.0	4.63	7.31	2.09	.41	-2.37*
SU II	49	10.5	6.53	4.90	2.92	.68	-1.71
SU III	49	10.1	5.52	5.12	3.11	.87	-0.28

** significant at .01 level

* significant at .05 level

DIFFERENCES IN ACHIEVEMENT ON EACH SUBUNIT BETWEEN THE SUMMATIVE AND POSTSUMMATIVE TESTS WITHIN THE MMM AND NNN GROUPS

	N	$\bar{X}$(ST) (max=10)	SD(ST)	$\bar{X}$(ST) (max=10)	SD(ST)	r	t
MMM SU I	51	6.75	2.86	7.67	2.65	.69	-3.03*
SU II	51	5.73	3.72	6.24	3.39	.84	-1.77
SU III	51	5.94	3.49	6.76	3.05	.84	-3.07*
NNN SU I	50	7.31	2.09	7.73	1.96	.56	-1.57
SU II	49	4.90	2.92	5.94	3.10	.78	-3.68*
SU III	49	5.12	3.11	6.35	2.98	.72	-3.84*

* significant at .01 level

ADJUSTED MEANS OF ACHIEVEMENT ON EACH SUBUNIT AS MEASURED DURING THE SUBUNIT, ON THE SUMMATIVE TEST AND ON THE POSTSUMMATIVE TEST

		GROUP	
		MMM	NNN
SU I	during SU	10.2	9.00
	on ST	4.46	5.13
	on PST	6.17	6.38
SU II	during SU	6.13	6.34
	on ST	2.49	1.90
	on PST	3.47	3.23
SU III	during SU	10.1	6.13
	on ST	2.70	2.03
	on PST	4.50	4.28

ADJUSTED MEANS ON ACHIEVEMENT ON UNIT AS MEASURED ON SUMMATIVE AND POSTSUMMATIVE TESTS

	GROUP	
	MMM	NNN
ST	9.56	8.74
PST	14.1	13.5